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WASHINGTON UNIVERSITY IN ST. LOUIS

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Essays on Financial and Labor Markets with Frictions

by

Feng Dong

A dissertation presented to the
Graduate School of Arts and Sciences
of Washington University in
partial fulfillment of the
requirements for the degree
of Doctor of Philosophy

May 2014

St. Louis, Missouri

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Acknowledgment

My dissertation would not have been finished without the invaluable support from my advisors. To start with, I would like to express my gratitude to my Chair, Stephen Williamson, who has been generously offering me constant guidance and encouragement since my second year of PhD study. Meanwhile, Costas Azariadis, Yongseok Shin and Yi Wen have kindly provided tremendous help since the early stages of these three chapters of the dissertation. The insightful feedback from all these advisors has broadened my research vision and pushed me to go as far as I can. I am also honored to receive helpful comments when I worked at the Research Division of Federal Reserve Bank of St. Louis and during my participation in academic conferences. In particular, I wish to thank Gaetano Antinolfi, Jinhui Bai, Yan Bai, Saki Bigio, Wei Cui, Bill Gavin, Boyan Jovanovic, Benham Lee, Rody Manuelli, B. Ravikumar, Juan M. Sánchez, Russell Tsz-Nga Wong, Pengfei Wang, Ping Wang, David Wiczer, and Tao Zha for their helpful comments. The usual disclaimer applies and all errors are mine. Special thanks go to Carissa Re, Karen Rensing, and Sonya Woolley for their administrative support. Finally, I would like to thank my parents, Zhixi Dong and Tongzhen Lin, and my fiancée, Yi Dong, for their love and infinite patience.

For my parents and for my fiancée

Abstract of Dissertation

Essays on Financial and Labor Markets with Frictions

by

Feng Dong

Doctor of Philosophy in Economics

Washington University in St. Louis, 2014

Professor Stephen Williamson, Chair

The dissertation, which consists of three chapters, is devoted to exploring financial and labor markets with frictions.

CHAPTER I: UNEMPLOYMENT AND CAPITAL MISALLOCATION. The recent recession was associated not only with a marked disruption in the credit market, but also a sharp deterioration in labor market conditions, as evidenced by high unemployment rates and an outward shift in the Beveridge curve. Motivated by such co-movements of the credit market and the labor market, in this chapter I develop a tractable dynamic model with heterogeneous entrepreneurs, credit constraints, and labor-search frictions. In this framework, the misallocation of capital across firms has an adverse effect on the matching efficiency in the labor market. I then quantify the importance of capital misallocation for understanding the behavior of unemployment rate. I find that the credit crunch was the key driving force behind the outward shift in the Beveridge curve during and after the Great Recession. More

broadly, I find that credit market frictions and labor search frictions almost equally contributed to unemployment over all business cycles between 1951 and 2011.

CHAPTER II: ASSET EXCHANGE WITH SEARCH FRICTIONS AND COSTLY INFORMATION ACQUISITION. The second chapter presents a model to characterize conditions under which centralized and decentralized markets (CM/DM) co-exist for asset trading. The asset payoff and trading motive are the seller's private information. CM is immune to search frictions, but suffers from adverse selection. In contrast, DM is subject to search frictions, but it is sustainable since buyers acquire costly information on the asset payoff, and offer a trading menu different from that posted by uninformed buyers. As matching efficiency in the DM increases and the information cost decreases, more trade migrates from CM with adverse selection to DM with search frictions. In the limit, DM with search frictions converges to CM with complete information. I use the model to address the heterogeneous welfare effect of a government asset purchase programs like the Troubled Asset Relief Program (TARP).

CHAPTER III: A SEARCH-BASED THEORY OF THE LIFE-CYCLE PATTERN OF ASSET HOLDING. The third chapter investigates the implications of search frictions for a household's life cycle pattern of asset trading as well as for its size distribution in the OTC. General types of preferences are considered and the usual search-theoretic restriction of indivisibility on asset holding is removed. I employ the birth-and-death process to analytically characterize the non-stationary life-cycle pattern of asset holding by each cohort. In the presence of search frictions in the OTC, our paper predicts that the life cycle of asset holding by each cohort conforms to a geometric distribution while the size distribution of asset holding in each cross-section follows a logarithmic pattern. In the end, our model yields Gibrat's law for asset trading in the OTC.

Chapter 1

Capital Misallocation and Unemployment

1.1 Introduction

The 2007-2008 financial crisis was accompanied by a marked increase in unemployment and a serious disruption in credit markets. First, the ratio of external funding to non-financial assets, a key measure used in the literature to characterize the functioning of the credit market, shrank significantly, as demonstrated in the right panel of Figure (1.1).¹ Second, as the left panel of Figure (1.1) shows, not only did the unemployment rate increase significantly over time, but the Beveridge curve also shifted outward beginning in the last quarter of 2008. Motivated by such co-movements of the credit market and the labor market, I develop a tractable dynamic model with heterogeneous entrepreneurs, credit constraints, and labor-search frictions. I find that the credit crunch was the key driving force behind the outward shift in the Beveridge curve during and after the Great Recession. More broadly, I find that credit market frictions and labor search frictions almost equally contributed to unemployment over all business cycles between 1951 and 2011.

I employ two layers of frictions to model the relationship between credit and labor markets. On the one hand, I introduce credit frictions by using a collateral constraint, which is a powerful tool to characterize credit crunches. On the other

¹The measure is considered in Buera and Moll (2013) and Buera, Fattal-Jaef and Shin (2013). Both non-financial corporate and non-financial non-corporate business in the Flow of Funds Accounts are considered. Details are documented in Appendix A.

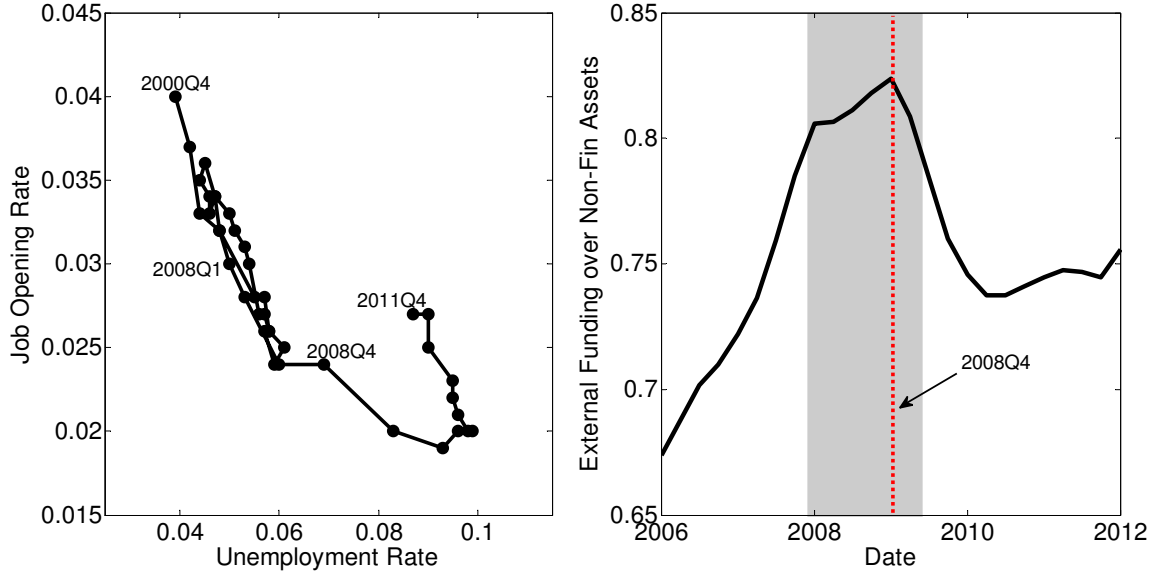


Figure 1.1: **Left Panel: Beveridge Curve, Job Openings and Labor Turnover Survey (JOLTS); Right Panel: External Funding over Non-Financial Assets of Non-Financial Business, Flow of Funds Accounts**

hand, I use competitive search to model equilibrium unemployment. Recent empirical findings by Davis, Faberman and Haltiwanger (2013) show that job-filling rates vary significantly across firms. However, a direct implication of random search is that job-filling rate is independent of firm's heterogeneous characteristics. As will be shown in our model, the prediction of competitive search is in line with the empirical regularity.

Entrepreneurs are heterogeneous in two dimensions, net worth and productivity. The former is endogenous and the latter is an exogenous stochastic process. There are three sources of aggregate shocks: i) a credit shock, *i.e.*, the tightening of collateral constraints in the credit market; ii) a matching shock, *i.e.*, the decrease of matching efficiency in the labor market; and iii) an aggregate productivity shock. When a credit crunch occurs, the collateral constraint tightens and more capital would have to be used by relatively unproductive entrepreneurs. The key theoretical contribution of this paper is that capital misallocation worsens labor misallocation, even though it is not accompanied by an adverse matching shock that directly disrupts the labor market.² Therefore credit imperfections contribute to endoge-

²Since our model involves capital misallocation, it belongs to the recently burgeoning literature on misallocation, which mainly includes Hsieh and Klenow (2009), Restuccia and Rogerson (2008), Bartelsman et al. (2012), and a recent discussion by Hopenhayn (2013), among others. Moreover, there has been extensive discussion on capital misallocation due to financial frictions, such as

nous matching efficiency in equilibrium and thus to shifts in the Beveridge curve. In addition to analytically illustrating the effect of capital misallocation on labor misallocation, I also show that equilibrium TFP is determined by the interaction between credit and labor frictions.³

The key transmission mechanism proceeds as follows. Although workers are homogeneous, the marginal value of being matched with labor increases with an entrepreneur's productivity. Therefore, entrepreneurs with heterogeneous productivity have an incentive to post different wage offers. I use competitive search to implement this idea. Entrepreneurs with higher productivity tend to post higher wage positions with more workers queuing for those jobs. Thus the job-filling rate will be higher for more productive entrepreneurs. In equilibrium, wage dispersion for homogeneous workers emerges with an endogenous set of segmented labor markets, as in standard competitive search models.

If there is a negative shock to the credit market, *i.e.*, the collateral constraint tightens, then capital misallocation worsens, since the interest rate decreases and more capital is used by relatively unproductive entrepreneurs. Since the job-filling rate in active sub-labor markets increases with an entrepreneur's productivity, the redistribution of capital from high-productivity to low-productivity firms decreases the total number of matched workers. In addition to the direct effect imposed on unemployment, capital misallocation also generates an indirect and offsetting effect in general equilibrium such that workers also move from labor markets with high productivity to those with lower productivity. Therefore, the job-filling rates as well as equilibrium wage dispersion in all sub-labor markets responds to credit crunches in general equilibrium. However, the concavity of the matching function in each active sub-labor market implies that job destruction by high-productivity entrepreneurs will outweigh job creation by low-productivity ones. Therefore those indirect general-equilibrium effects are dominated by the direct effect described above. In sum, this is how credit crunches contribute to the outward shift in the Beveridge curve.⁴

Buera, Kaboski and Shin (2011), Azariadis and Kaas (2012), Moll (2012), Wang and Wen (2012), Bigio (2013), Buera and Moll (2013), Cui (2013), Khan and Thomas (2013), and Liu and Wang (2013).

³Lagos (2006) develops a model of TFP with labor search frictions. Our work contributes to this line of literature by incorporating both credit and labor search frictions into an otherwise standard RBC model.

⁴Complementary to our work, Mehrotra and Sergeyev (2012) develop a multi-sector model with labor search to characterize conditions under which sector-specific shock, such as in the construction sector, can decrease aggregate matching efficiency and generate an outward shift in the Beveridge

In each period, the collateral constraint is not necessarily binding for all heterogeneous entrepreneurs. An infinite-horizon model with this setup is potentially complicated. Moreover, I allow for capital accumulation with both financial frictions in the credit market and search frictions in the labor market. Our model is highly tractable because of the linearity of individual policy functions, which is driven by the linearity of the capital revenue in equilibrium. The analytical solution is beneficial in making transparent the mechanism through which capital and labor misallocation interact with each other.

The unemployment effect of capital misallocation is not only of theoretical interest, but also offers a new channel for amplification and propagation in our quantitative analysis. A negative credit shock not only creates capital misallocation and works at the intensive margin, but also affects the extensive margin by lowering matching efficiency. Therefore, even in the absence of the price effect in Kiyotaki and Moore (1997), credit frictions have an amplification effect with a new channel through which capital misallocation worsens labor misallocation. When it comes to the unemployment effect, credit crunches lower endogenous matching efficiency in the labor market. Additionally, the new amplification effect of credit crunches dampens capital accumulation and thus further increases unemployment and lowers output in the next period. This is a dynamic implication of credit crunches for aggregate variables of interest.

I then move on to quantify the unemployment effect of credit imperfections as well as that of labor search frictions. In particular, I explore how much credit and labor frictions explain unemployment. Moreover, does the credit crunch contribute to the outward shift in the Beveridge curve in the recent financial crisis? Three insights are gained from the quantitative exercise. First and most importantly, the counter-factual analysis shows that the credit crunch serves as a driving force behind the outward shift in the Beveridge curve in the recent financial crisis. I present a preview in Figure (1.2). The left panel indicates that the Beveridge curve predicted by our model fit well with the data. The right panel illustrates that, if there had been no credit crunch in the last quarter of 2008, the predicted unemployment would continue to rise with the negative shocks to aggregate productivity and to the matching efficiency in the labor market. However, in the absence of the credit crunch, the predicted Beveridge curve would not shift outward, but instead would move along with the original curve prior to the financial crisis.

curve.

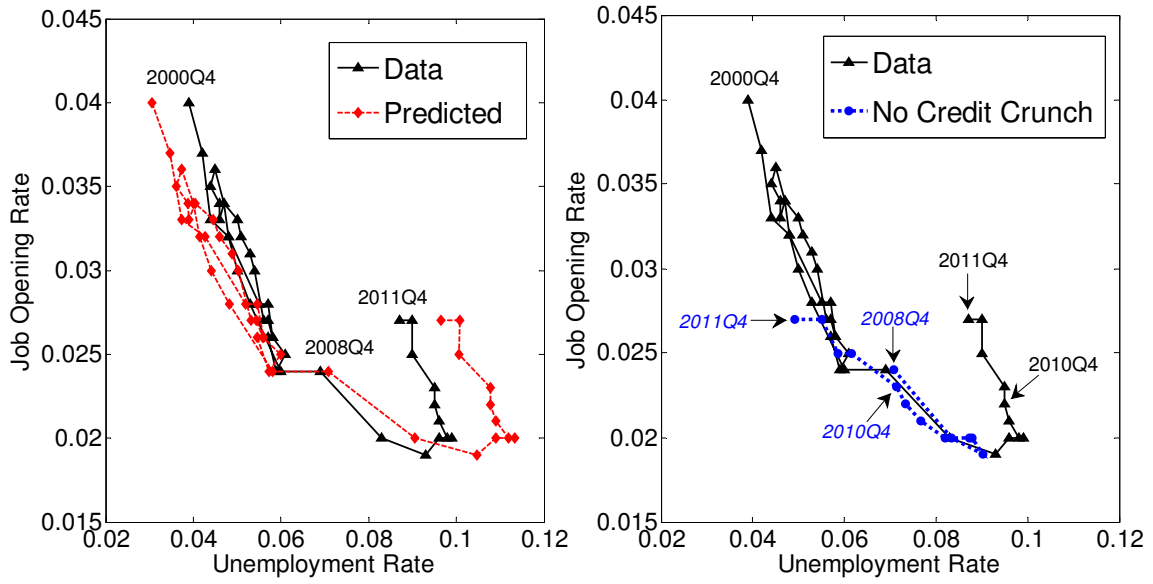


Figure 1.2: **Left Panel: Data and Model-Predicted for the Beveridge Curve; Right Panel: Data and Model-Predicted without the Credit Crunch in 2008**

The second finding of our quantitative exercise shows that the shocks to the credit or labor markets generate a co-movement on output and unemployment. This prediction is in line with the data prior to the recent three recessions. In contrast, the shock to aggregate productivity generates a gap between output and unemployment recovery. This is what happened in the past three recessions. This phenomenon is called a jobless or sluggish recovery and has spawned a large literature; see Berger (2012), among others. Most of the literature assumes a frictionless labor market and only addresses the recovery gap between output and employment numbers. Therefore previous studies cannot explain the persistently high unemployment rates of the past recessions.⁵ Finally, I also find that the shock to the credit market and the shock to the labor market increases and decreases respectively the power of credit imperfections in explaining unemployment. Since both credit and labor shocks are procyclical, the contribution of credit imperfections to unemployment could be ambiguous in theory. Confronting the model with data after a calibration to the US economy indicates that the explanatory power of credit imperfections is procyclical. That is, the labor market itself receives a relatively larger negative shock in recessions. The decomposition exercise suggests credit imperfections account for around 46% of unemployment over all cycles.

⁵Jaimovich and Siu (2013) are an exception. They investigate the empirical relationship between jobless recoveries and job polarization, and then set up a labor search model with equilibrium unemployment.

In addition to investigating the aggregate implications of three shocks of interest, tractability also offers a transparent discussion on the different micro-level implications of these shocks. I test the predictions of different shocks with micro-level empirical findings. Credit shocks are seemingly most essential in explaining the widening productivity dispersion as well as the disproportional employment loss of firms with different sizes. I generalize the transmission mechanism through which capital misallocation worsens labor misallocation. I begin by introducing a general tax scheme upon capital revenue, which treats the baseline as a special case. I then put an additional constraint on working capital to our model, which generates a non-trivial labor wedge in equilibrium. Finally, I show that endogenizing firm's search effort amplifies the transmission channel in the baseline.

The recent financial crisis has spawned a large volume of research on the role financial shocks play in output fluctuation, following the works of Williamson (1987), Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), and Bernanke, Gertler and Gilchrist (1999). Jermann and Quadrini (2012) and Khan and Thomas (2013) are two such recent studies. However, very few papers connect financial frictions and unemployment.⁶ Wasmer and Weil (2004) adopt matching functions with random search to model frictions in both credit and labor markets.⁷ They then use the general-equilibrium interaction between these two markets to illustrate the workings of a financial accelerator. Monacelli, Quadrini and Trigari (2011) discuss the role of credit frictions in unemployment by introducing the strategic use of debt by firms with limited enforcement.⁸ They build the model to explain why firms lower labor demand after a credit contraction even though there is no shortage of funds for hiring. Miao, Wang and Xu (2013) integrate an endogenous credit constraint into a model with random search. They show that the collapse of the bubble, one of the self-fulfilling equilibria, tightens the credit constraint, and in turn decreases labor demand. Liu, Miao and Zha (2013) incorporate the housing market and the labor market in a DSGE model with credit and search frictions. They then make a structural analysis of the dynamic relationship between

⁶Merz (1995) and Andolfatto (1996) were among the first to introduce labor search frictions in the RBC framework, which admits capital accumulation but is subject to no financial frictions. See Shimer (2010) for a survey on the recent development of quantitative analysis for labor search.

⁷A quantitative extension is done by Petrosky-Nadeau and Wasmer (2013), among others. Meanwhile, see Carrillo-Tudela, Graber, and Waelde (2013) for a recent related theoretical model.

⁸Garin (2013) and Blanco and Navarro (2013) extend the work of Monacelli, Quadrini and Trigari (2011) by allowing for capital accumulation and by introducing flexible number of employees and equilibrium default, respectively.

land prices and unemployment. All of the aforementioned papers focus on the connection between firm-side credit imperfections and unemployment, while Bethune, Rocheteau and Rupert (2013) emphasize the relationship between household credit and unemployment.

Our paper complements the work of Buera, Fattal-Jaef and Shin (2013). Both papers quantify the effect of a credit crunch on unemployment in a heterogeneous-entrepreneurs model with credit frictions and employment frictions. However, our papers differ in several important dimensions. First, their analysis is largely quantitative while the linear property of our model generates tractability and makes transparent the new channel contributed by our paper. Second, we use different modeling strategies for equilibrium unemployment. They specify a Walrasian labor market with a unique and publicly displayed price. To sustain equilibrium unemployment, they assume only a fraction of unemployed workers can enter the centralized hiring market in a given period. I instead use competitive search by following Shimer (1996) and Moen (1997). Finally, they focus on the recent credit crunch while I take into account the historical business cycles as well as the recent recession.

The rest of the paper is organized as follows. Section 2 describes the model setup. Section 3 characterizes general equilibrium. Section 4 presents a quantitative analysis. Section 5 addresses the disaggregate implications of our model with recent micro-level empirical findings. Section 6 concludes. Appendix A provides the data definition, description and calculation. Appendix B offers a simplified and static model. Appendix C considers model extension. Appendix D includes all omitted proofs.

1.2 Model

This section describes the model setup by introducing agents and specifying frictions in credit and labor markets.

1.2.1 Demography and Timing

Time is discrete and goes from zero to infinity. There is no information asymmetry. The economy is populated by three kinds of infinitely lived players: workers,

entrepreneurs and financial intermediaries.⁹

Workers. There is a representative household with measure L of homogeneous household members. Each worker has one unit of indivisible labor. I assume the household has access to neither production skills nor the credit market. If a worker is unemployed, she has no revenue.¹⁰ If a worker is matched with an entrepreneur, she receives labor revenues after production.¹¹ The household distributes consumption equally to each member by pooling labor revenue at the end of each period. All workers engage in hand-to-mouth consumption. In this paper, the new channel through which capital misallocation affects unemployment is on the side of labor demand. To sharpen our transmission mechanism, I assume labor supply is inelastic.¹²

Entrepreneurs. There is a unit measure of entrepreneurs. Only entrepreneurs have access to the credit market as well as to production skills. Entrepreneurs are heterogeneous in two dimensions: one is net worth a while the other is productivity x . I assume x is the product of aggregate productivity z and individual component φ , *i.e.*, $x = z \cdot \varphi$. The distribution of net worth endogenously evolves over time while that of an idiosyncratic and aggregate productivity shock is exogenous. The distribution of individual productivity is denoted as $F(\cdot)$ with a bounded support $[\underline{\varphi}, \bar{\varphi}]$. In the next period, individual productivity φ is preserved or is re-drawn from some fixed distribution $\tilde{F}(\cdot)$ with probability ρ and $1 - \rho$, respectively. When $\rho = 1$, it is degenerate to the case with *iid* productivity shock. For simplicity, I assume $\tilde{F}(\cdot)$ coincides with $F(\cdot)$ in the first period. Therefore, the distribution of individual productivity is stationary over time.¹³ The stochastic process governing z is not essential for our analysis right now. I will return to it in the quantitative

⁹Our paper does not consider occupational choice. See Wiczer (2012) and Buera, Fattal-Jaef and Shin (2013), among others, for a quantitative discussion on unemployment with occupational choice.

¹⁰That is, I assume the replacement ratio is zero throughout this paper. As shown soon, I assume a fixed labor supply and focus on the demand side for labor. Thus this assumption of no unemployment compensation does not affect the key channel of our paper. However, as pointed out in the quantitative analysis by Hobijn and Sahin (2012) and Hagedorn, Karahan, Manovskii and Mitman (2013) with a different context of modeling, the extension of unemployment insurance benefits could be quantitatively important in explaining the worsening labor market in the past recession.

¹¹There is no constraint on working capital in the baseline model. Appendix C considers the case in which entrepreneurs need to pay part of wage bill before production.

¹²Alternatively, I can explicitly specify the household's utility function as $U_W = \mathbb{E} \left\{ \sum_{t=0}^{\infty} \beta^t \cdot \left[\log(C_t) - \xi \cdot \frac{L_t^{1+\nu}}{1+\nu} \right] \right\}$, where C and L denotes consumption and labor supply respectively. Since the household has a continuum of workers and does not save, I have $C = W \cdot L$, where W denotes expected labor revenue. The details of labor search and matching is specified very soon in the part of labor market. The log-utility setup, alongside with the first order condition of the intra-period decision on labor supply, implies a fixed labor supply by the household.

¹³In general, I have $F_{t+1}(\cdot) = \rho \cdot F_t(\cdot) + (1 - \rho) \cdot \tilde{F}(\cdot)$.

analysis. For tractability, I assume productivity shock is independent of net worth. Therefore the joint distribution $H(a, \varphi)$ can be rewritten as the product of $F(\varphi)$ and $G(a)$, the distribution of individual productivity and that of net worth. An entrepreneur's objective function is given by

$$U_E = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \cdot \log(c_t) \right],$$

where c_t denotes consumption.

Financial Intermediary (FI) & Credit Market. The representative financial intermediary is risk neutral and fully competitive. I assume all borrowing and lending between entrepreneurs is intermediated by FI. One of the possible elements to make FI essential is to assume FI can verify an entrepreneur's individual productivity but it is too costly for entrepreneurs themselves if they directly contact each other. FI herself does not own, produce or use capital.¹⁴ I model credit imperfections by assuming productive entrepreneurs cannot borrow as much as they want.

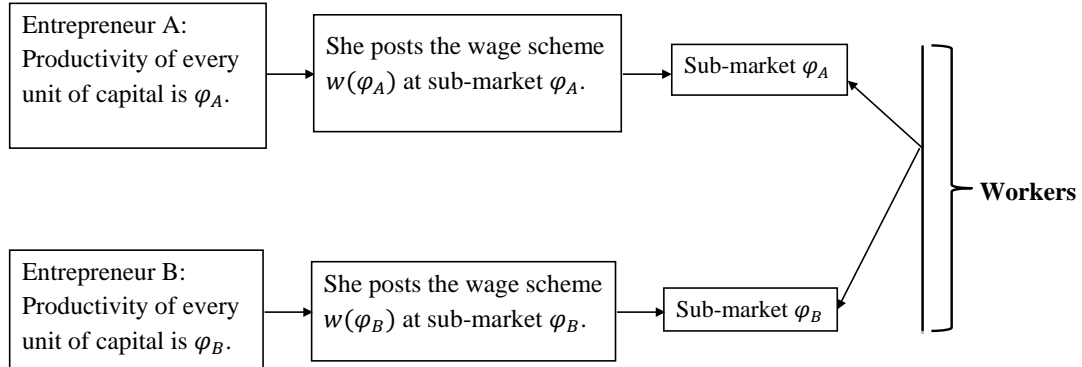


Figure 1.3: **Wage Posting by Active Entrepreneurs**

Labor Market. I use competitive search, which is also called directed search, to model equilibrium unemployment. As is standard in the literature, the production function is Leontief: only after one unit of capital from entrepreneur- (a, φ) is matched with one unit of labor can φ units of consumption goods be realized. Entrepreneur- (a, φ) could either borrow and produce by posting a wage contract $w(\varphi)$ in sub-market

¹⁴Dong and Wen (2013) address a case in which FI not only intermediates borrowing and lending, but also produces capital goods with a linear transformation technology.

φ , or lend to other entrepreneurs in the credit market.¹⁵ The opportunity cost of running capital is the endogenous interest rate r .¹⁶ Therefore, not all entrepreneurs choose to produce. If a worker goes to sub-market φ and gets matched, she obtains wage $w(\varphi)$. Workers self-select into active sub-markets $\varphi \in \Phi^A \subseteq \Phi$. See Figure (1.3). Only matched workers receive revenues. The household pools all the labor income together and distributes it equally to all members. Each household member engages in hand-to-mouth-consumption. The borrower entrepreneurs receive capital revenue, part of which they pay back to lender entrepreneurs via the financial intermediary. All entrepreneurs make decisions about consumption and saving.

State Variables and Timing. I assume all matched relationships between firms and workers are terminated at the end of every period. This assumption simplifies our analysis. If I use a long-term contract, then entrepreneurs would be heterogeneous in three dimensions in each period: net worth, productivity, and numbers of employed workers. In turn, the theoretical analysis would be deprived of tractability.¹⁷ Therefore I make the above assumption.¹⁸ Consequently, the idiosyncratic state variable is two dimensional, (a, φ) , the net worth and productivity. The aggregate state is denoted as $X = (z, \lambda, \eta, H(a, \varphi))$, where z is aggregate productivity shock, λ is the shock to the credit market, η is the matching efficiency in every sub-labor market, and $H(a, \varphi)$ is the joint distribution of net worth and productivity. Given our assumption about the productivity shock, the aggregate state can be rewritten as $X = (\lambda, \eta, F(x), G(a))$, where $F(\varphi)$ and $G(a)$ denote the distribution of productivity and that of net worth, respectively, and the product yields their joint distribution. Finally, I present the time-line in Figure (1.4).

1.2.2 Labor Market

As is standard in the literature, the matching function $m(v(\varphi), l(\varphi))$ in all sub-markets $\varphi \in \Phi$ is homogeneous of degree one, and increases with both arguments,

¹⁵The framework of competitive search implies $w(\varphi)$ has nothing to do with productivity distribution. This in turn helps preserve model tractability.

¹⁶Since there is no entry and exit, I assume for simplicity that there is no explicit cost of wage posting.

¹⁷Schaal (2012) characterizes and quantifies a search model with heterogeneity in productivity and labor use. However, there is heterogeneity in net worth since there is no capital use and capital accumulation. As noted at the end of Schaal (2012), it is promising and challenging to consider financial frictions after introducing capital accumulation. Complementary to his work, our paper considers heterogeneity in productivity and capital.

¹⁸However, this assumption immediately implies the ratio of job destruction to total employment is 100%. To solve this problem, I use the net flow to measure job destruction and job creation. See more details in Section 4.

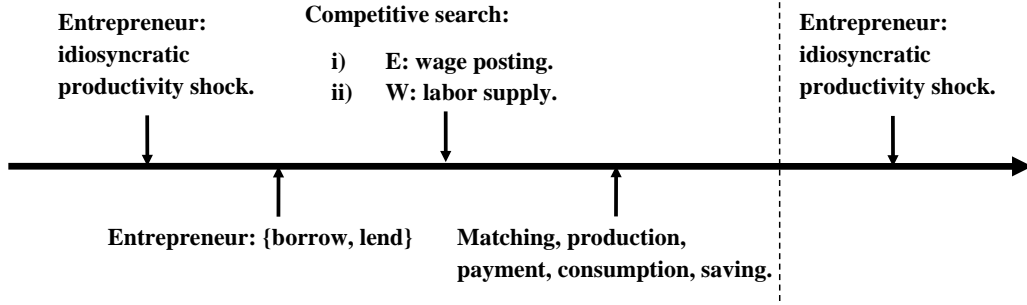


Figure 1.4: **Time-line**

where $v(\varphi)$ and $l(\varphi)$ denote, respectively, the measure of capital and labor with market tightness $\theta(\varphi) \equiv l(\varphi)/v(\varphi)$. Then the job-filling rate and job finding rate, $q(\theta(\varphi))$ and $p(\theta(\varphi))$, have the following property: $q' > 0$, $q'' < 0$, $p' < 0$ and $p'' > 0$, where

$$\begin{aligned} q(\theta(\varphi)) &\equiv \frac{m(v(\varphi), l(\varphi))}{v(\varphi)} = m(1, \theta(\varphi)) \\ p(\theta(\varphi)) &\equiv \frac{m(v(\varphi), l(\varphi))}{l(\varphi)} = m\left(\frac{1}{\theta(\varphi)}, 1\right) = \frac{q(\theta(\varphi))}{\theta(\varphi)}. \end{aligned}$$

I assume throughout the paper that the matching function is Cobb-Douglas, *i.e.*, $m(v(\varphi), l(\varphi)) = \eta \cdot v(\varphi)^\gamma \cdot l(\varphi)^{1-\gamma}$ with $\gamma \in (0, 1)$, where η denotes matching efficiency and is exogenously given.¹⁹ Due to search frictions and heterogeneity in capital productivity, there exists no unique wage such that labor supply equals demand. Instead, I only have the following constraint on labor supply.

$$\int_{\Phi} l(\varphi) \cdot d\varphi = L. \quad (1.1)$$

I formulate $\pi(\varphi, W)$, the expected revenue of one unit of capital in sub market- φ , as below.

$$\pi(\varphi, W) \equiv \max_{\{\theta(\varphi, W), w(\varphi, W)\}} \{q(\theta(\varphi, W)) \cdot (\varphi - w(\varphi, W))\}, \quad (1.2)$$

subject to

$$p(\theta(\varphi, W)) \cdot w(\varphi, W) = W, \quad (1.3)$$

where $W(\varphi) = W(\varphi') \equiv W$ denotes the expected wage revenue by going to sub-market $\varphi, \varphi' \in \Phi^A \subseteq \Phi$, where Φ^A denotes the set of entrepreneurs active in production. I characterize Φ^A in Section 2.4, and right now treat it as given. I now characterize the endogenous wage offer in active sub markets Φ^A .

¹⁹Motivated by recent empirical findings, Appendix C endogenizes firms' recruiting efforts, which amplifies the transmission mechanism in the baseline.

Proposition 1. (Wage Scheme)

1. Given W , the market tightness in any active sub-market $\varphi \in \Phi_A$ is determined by

$$q'(\theta(\varphi)) = \frac{W}{\varphi}. \quad (1.4)$$

2. The wage scheme and expected capital revenue obtained from sub-market $\varphi \in \Phi_A$ is given by

$$\begin{aligned} w(\varphi, W) &= \frac{W}{p(\theta(\varphi))} \\ \pi(\varphi, W) &= [q(\theta(\varphi)) - \theta q'(\theta(\varphi))] \cdot \varphi. \end{aligned} \quad (1.5)$$

3. Comparative statics:

$$\frac{\partial \pi(\varphi, W)}{\partial \varphi} > 0, \quad \frac{\partial \pi(\varphi, W)}{\partial W} < 0, \quad \frac{\partial \theta(\varphi, W)}{\partial \varphi} > 0, \quad \frac{\partial \theta(\varphi, W)}{\partial W} < 0, \quad \frac{\partial q(\theta(\varphi, W))}{\partial \varphi} > 0, \quad \frac{\partial q(\theta(\varphi, W))}{\partial W} < 0.$$

The marginal value of being matched with labor increases with the productivity. Therefore, the wage scheme increases with productivity. In turn, entrepreneurs with higher productivity enjoy a higher job-filling rate. Thus, high-productivity entrepreneurs are more efficient in both extensive and intensive margins. This observation is the key to understanding the general-equilibrium effect of capital misallocation on unemployment in the next section. Finally, Proposition 1 shows the expected capital revenue increases with productivity. This property, like that in Melitz (2003), delivers a cut-off point for active entrepreneurs and greatly simplifies our analysis in Section 2.4.

1.2.3 Entrepreneur

At the beginning of each period, entrepreneurs rely on two pieces of public information to decide whether or not to be active in production. One is the individual state variable, which includes net worth a and productivity φ . The other one is the aggregate state variable $X = (\lambda, \eta, z, F(\varphi), G(a))$. Assume some entrepreneur uses k units of capital for production. Then I use $b \equiv k - a$ to denote the external funding. That $b < 0$ means net lending. Since the production function is Leontief, active entrepreneurs posts their wage scheme $w(\varphi)$ for every unit of capital at sub market $\varphi \in \Phi_A$. For notational ease, I replace $\pi(\varphi, W)$ with $\pi(\varphi)$ in the rest of

the paper. Assume the law of large numbers holds here. Then the total capital revenue is $\Pi(k, \varphi) = \pi(\varphi) \cdot k$ for the entrepreneur with productivity φ and using k units of capital for production. I model credit frictions with the simplest collateral constraint, *i.e.*, $k \leq \lambda \cdot a$, where k and a denotes the total capital available and own net worth, respectively, and λ the exogenous financial shock to the credit market. If $\lambda = 1$, the credit market collapses and entrepreneurs are in autarky. If $\lambda = \infty$, the credit market is complete since the collateral constraint would never be binding. Finally, the constrained optimization of entrepreneur- (a, φ) is formulated as below.

$$V(a, \varphi; X) = \max \{ \log(c) + \beta \cdot \mathbb{E}[V(a', \varphi'; X') | X] \} \quad (1.6)$$

subject to

$$r \cdot b + c + i = \Pi(k, \varphi) = \pi(\varphi) \cdot k \quad (1.7)$$

$$a' = (1 - \delta) \cdot a + i \quad (1.8)$$

$$b = k - a \quad (1.9)$$

$$k \leq \lambda \cdot a \quad (1.10)$$

$$k \geq 0 \quad (1.11)$$

Equation (1.7) is the budget constraint with $\Pi(k, \varphi)$ being the capital revenue, $r \cdot b$ the debt repayment, c the consumption and i the investment for next period. Equation (1.8) is the accounting identity on investment, net worth and the total capital obtained for production. Equation (1.9) is the definition on external funding b . Equation (1.10) is a collateral constraint, in which the maximum available capital is proportional to the entrepreneur's own net worth. The collateral constraint $k \leq \lambda \cdot a$ implies the leverage ratio is the same across heterogeneous entrepreneurs, and has nothing to do with the interest rate r . This is purely for tractability.²⁰ As emphasized by Moll (2012), it is the linearity of collateral constraint that guarantees tractability. Equation (1.11) denotes a no-short-selling constraint.

I use the simplest form of collateral constraint. Unlike Kiyotaki-Moore (1997), I eliminate the price effect. As shown in Section 3, this simplification will illustrate the unemployment effect of capital misallocation in a transparent way. Moreover, I can anticipate that the additional consideration of price effect would strengthen the new channel proposed there. Second, credit imperfections are characterized by the above collateral constraint in a reduced-form way. There are several alternatives

²⁰I also tried a complicated version in which the collateral constraint is related to interest rate and productivity heterogeneity. The result is still tractable at both micro and aggregate levels. It is available upon request.

with micro-foundation to support the linear form of collateral constraint. In addition to the limited liability proposed by Kiyotaki and Moore (1997), I can also obtain the linearity by considering costly state verification by Williamson (1987) and Bernanke and Gertler (1989), or moral hazard by Holmstrom and Tirole (1997). Finally, our baseline only takes into account credit frictions and labor search frictions. This helps us focus on the unemployment effect of worsening capital misallocation in the simplest and most clear way.

1.2.4 Credit Market

I use this part to characterize the conditions under which the collateral constraint is binding for entrepreneurs heterogeneous in net worth and productivity. Denote $\Pi(k, \varphi)$ as the capital revenue by entrepreneurs with productivity φ and using k units of capital for production. Based on Proposition 1 and assuming the law of large number applies, I know the capital revenue is linear in k , and

$$\Pi(k, \varphi) = \pi(\varphi) \cdot k = kq(\varphi)\varphi - k\theta(\varphi)W, \quad (1.12)$$

Then the constrained optimization by entrepreneur- (a, φ) can be simplified as below.

$$V(a, \varphi; X) = \max \{ \log(c) + \beta \cdot \mathbb{E}[V(a', \varphi'; X') | X] \}$$

subject to

$$\begin{aligned} c + a' &= [r + (1 - \delta)] \cdot a + \max \{ \pi(\varphi) - r, 0 \} \cdot k \\ k &\in [0, \lambda \cdot a], \quad \lambda \in (1, \infty) \end{aligned}$$

The entrepreneur- (a, φ) can always receive the capital revenue $[r + (1 - \delta)] \cdot a$ by making a deposit to the financial intermediary. Additionally, if the entrepreneur uses k units of capital for production, then the net gain is $\pi(\varphi) - r$, where $\pi(\varphi)$ and r denotes the expected revenue and the opportunity cost of using one unit of capital for production. Therefore, the option value for each unit of capital held by an entrepreneur with productivity φ is $\max \{ \pi(\varphi) - r, 0 \}$. In turn, I follow Buera and Moll (2013) to define the return premium as $RP \equiv \mathbb{E}[\max(\pi(\varphi) - r, 0)]$. If there is no credit friction or no productivity heterogeneity, then the return premium is simply zero. Given the individual capital demand $k(\varphi, a)$, the clearing condition in the credit market is then obtained by

$$\int \int k(\varphi, a) \cdot h(\varphi, a) d\varphi da = \int \int a \cdot h(\varphi, a) d\varphi da. \quad (1.13)$$

I then use the following lemma to characterize the individual capital demand.

Lemma 1. (Capital Demand and Cash Holding) Capital demand by entrepreneur-
(a, φ) conforms to a corner solution, *i.e.*,

$$k(\varphi, a) = \begin{cases} 0 & \text{if } \varphi \in [\underline{\varphi}, \widehat{\varphi}] \\ \lambda \cdot a & \text{if } \varphi \in [\widehat{\varphi}, \overline{\varphi}] \end{cases},$$

where the cut-off value $\widehat{\varphi}$ is determined by

$$\pi(\widehat{\varphi}) = r, \quad (1.14)$$

and the ratio of cash holding to assets is $\lambda \cdot [1 - q(\varphi)]$.

Denote the aggregate net worth as $K \equiv \int a \cdot dG(a)$. The above lemma suggests the measure of capital in sub market φ is

$$v(\varphi) = \left[\int k(\varphi, a) dG(a) \right] \cdot f(\varphi) \cdot \mathbf{1}_{\{\varphi \geq \widehat{\varphi}\}} = \lambda K f(\varphi) \cdot \mathbf{1}_{\{\varphi \geq \widehat{\varphi}\}}. \quad (1.15)$$

Entrepreneurs with high enough productivity produce and hit a binding collateral constraint. The rest prefer lending in the credit market. The property of choosing corner solutions is due to the linearity of capital gains. Besides, this lemma immediately reveals that the set of active entrepreneurs is $\Phi_A = \{\varphi | \varphi \geq \widehat{\varphi}\}$. It is worth noting that, although active entrepreneurs want to borrow as much as they want with a binding collateral constraint, the equilibrium leverage ratio used for production is $\lambda \cdot q(\varphi)$ rather than λ in the presence of labor search frictions. Consequently, cash holding emerges in equilibrium. The ratio of cashing hold to assets decreases with productivity. This is determined by the use of capital with labor search frictions, which is illustrated as follows.

Corollary 1. (Double Selection on Capital Use) The productivity distribution of active entrepreneurs and that of matched entrepreneurs are

$$F^A(\varphi) = \frac{F(\varphi) - F(\widehat{\varphi})}{1 - F(\widehat{\varphi})}, \quad F^M(\varphi) = \frac{\int_{\widehat{\varphi}}^{\varphi} q(\varphi') \cdot dF(\varphi')}{\int_{\widehat{\varphi}}^{\overline{\varphi}} q(\varphi') \cdot dF(\varphi')},$$

and $F^M(\varphi) < F^A(\varphi) < F(\varphi)$.

It is worth noting that the equilibrium productivity distribution is $F^M(\varphi)$ rather than $F^A(\varphi)$. The latter is the truncated distribution in the first step. As proved in

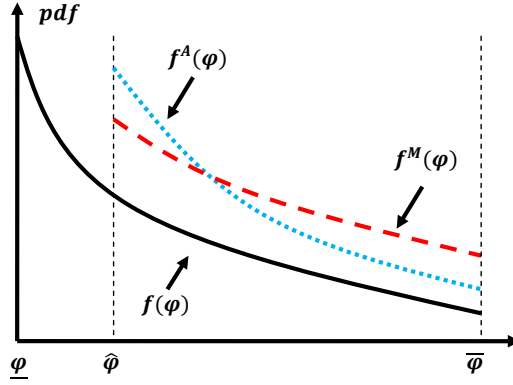


Figure 1.5: **Double Selection of Capital Use**

Proposition 1, the job-filling rate of active entrepreneurs increases their individual productivity. As a result, the equilibrium productivity distribution is obtained after the selection in the second step, which reflects in the weight $q(\varphi)$ in the above equation of $F^M(\varphi)$. I illustrate the relationship of these three distributions in Figure (1.5). In the end, I obtain the policy function of entrepreneur- (a, φ) in partial equilibrium.

Corollary 2. (Individual Policy Function) Given the aggregate state variable X , the consumption and saving by entrepreneur- (a, φ) is linear with her own net worth.

$$\begin{aligned} a_{t+1}(a_t, \varphi_t) &= \beta \cdot \Psi_t(\varphi) \cdot a_t \\ c_t(a_t, \varphi_t) &= \Psi_t(\varphi) \cdot a_t - a_{t+1}(a_t, \varphi_t), \end{aligned}$$

where $\Psi_t(\varphi) \equiv \lambda_t \cdot \max\{\pi_t(\varphi) - r_t, 0\} + [r_t + (1 - \delta)]$.

The linearity of policy function admits a tractable aggregation.²¹ Therefore, I can keep track of the endogenous evolution of the distribution without resorting to purely numerical work like Krusell and Smith (1998). The linear property of policy function makes it easy for us to connect with recent literature on credit frictions. For example, Wang and Wen (2012) develop an incomplete credit market model with heterogeneity in investment efficiency as well as with partial irreversibility such that $a' \geq \lambda_I \cdot (1 - \delta) \cdot a$. Notice that $\lambda_I = 0$ and $\lambda_I = 1$ denote the cases with perfect reversibility and complete irreversibility, respectively. Based on the above corollary, the individual policy function is still tractable with the additional

²¹In the presence of partial irreversibility, the policy function is adjusted as $a_{t+1}(a_t, \varphi_t) = \max\{\beta \cdot \Psi_t(\varphi), \lambda_{I,t} \cdot (1 - \delta)\} \cdot a_t$. Thus the linearity property is preserved.

constraint of partial investment irreversibility upon our framework. In this scenario, the intertemporal decision would be adjusted as

$$a_{t+1}(a_t, \varphi_t) = \max \{ \beta \cdot \Psi_t(\varphi), \lambda_I \cdot (1 - \delta) \} \cdot a_t.$$

1.3 Equilibrium

I have so far addressed the decisions of all agents in partial equilibrium. I summarize the key results in Figure (1.6).

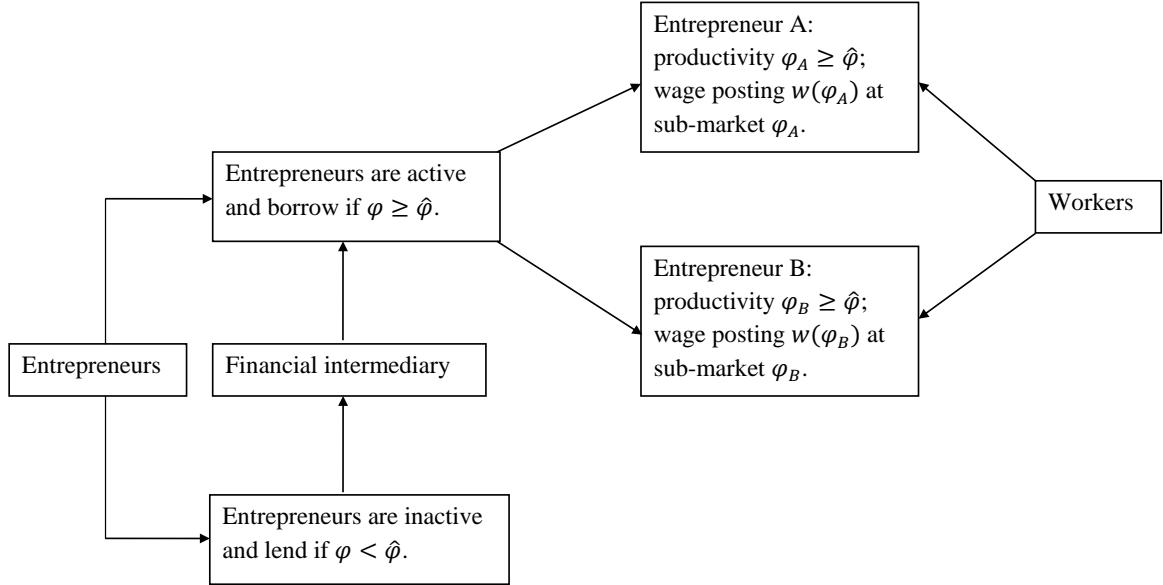


Figure 1.6: **Decision Rules of All Agents**

This section is devoted to exploring the general equilibrium of our model with heterogeneous entrepreneurs, and with credit and labor search frictions. I characterize not only the equilibrium in each period, but also the transition dynamics. I start with defining the recursive competitive equilibrium as below.

Definition 1. (Recursive Competitive Equilibrium) A recursive competitive equilibrium consists of

1. labor supply $l(\varphi)$, capital $v(\varphi)$ and market tightness $\theta(\varphi)$ at active sub-market $\varphi \in \Phi_A$,
2. a set of price functions, including the interest rate r , the wage scheme $w(\varphi)$ and the expected labor gain from sub-market $W(\varphi)$ in active sub-market $\varphi \in \Phi_A$,
3. a set of individual policy functions, including consumption c , debt b , and net worth for next period a' ,
4. the value function $V(a, \varphi)$,
5. the law of motion for the aggregate state variable $X = (z, \lambda, \eta, F(\varphi), G(a))$, such that,
 - given X and W the market tightness $\theta(\varphi) = l(\varphi)/v(\varphi)$ is determined by Equation (1.4), $v(\varphi)$ by Equation (1.15) and wage $w(\varphi)$ by Equation (1.5),
 - given X , the cut-off point, $\widehat{\varphi}$, the interest rate r , and the expected wage revenue W are jointly determined by Equations (1.14), (1.13), and (1.1),
 - $c(a, X)$ and $a'(a, X)$ is the solution to the entrepreneur's dynamic optimization, and the value function $V(a, X)$ is obtained with $c(a, X)$ and $a'(a, X)$,
 - the credit market clears as in Equation (1.13).

1.3.1 Equilibrium Wedges

I first address the social planner's problem. More specially, there is only labor search friction in the benchmark. Then the problem is formulated as below.

$$Y^* = \max_{\{v(\varphi), l(\varphi)\}} \int_{\Phi} z \cdot \varphi \cdot m(v(\varphi), l(\varphi)) d\varphi$$

subject to

$$\begin{aligned} \int_{\Phi} v(\varphi) d\varphi &\leq K \equiv \int \int a \cdot h(\varphi, a) d\varphi da \\ \int_{\Phi} l(\varphi) d\varphi &\leq L \\ v(\varphi), l(\varphi) &\geq 0, \end{aligned}$$

where $v(\varphi)$ and $l(\varphi)$ denotes the measure of capital and labor in sub-labor market φ . I summarize the key results below.

Lemma 2. (Benchmark) If the matching function is constant return to scale, the most efficient allocation is that all capital and labor are assigned to the most productive entrepreneurs, *i.e.*, $v^*(\varphi) = K \cdot \mathbf{1}_{\{\varphi=\bar{\varphi}\}}$, $l^*(\varphi) = L \cdot \mathbf{1}_{\{\varphi=\bar{\varphi}\}}$, $Y^* = z \cdot \bar{\varphi} \cdot m(K, L)$, $N^* = m(K, L)$, $u = 1 - \frac{N^*}{L}$, and $ALP^* \equiv \frac{Y^*}{N^*} = z \cdot \bar{\varphi}$.

First, the efficient allocation can be realized if all firms have to post a unique wage. The Bertrand competition would then drive up the wage to $z \cdot \bar{\varphi}$. Second, the benchmark results on allocation have a caveat. If I use the span-of-control model by Lucas (1978), then it is not necessarily true that all resources should be used by the most productive firms.

In the rest of this section, I characterize the equilibrium allocation of the decentralized economy. To start with, I make the below assumption.

Assumption 1. $\Upsilon(\tilde{\varphi}) \equiv \frac{\mathbb{E}_F\left(\varphi^{\frac{1-\gamma}{\gamma}} \mid \varphi \in [\tilde{\varphi}, \bar{\varphi}]\right)}{\left[\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\tilde{\varphi}, \bar{\varphi}]\right)\right]^{1-\gamma}}$ strictly increases with $\tilde{\varphi} \in (\underline{\varphi}, \bar{\varphi})$ for $\gamma \in (0, 1)$

This assumption is reasonable in the sense that it is held with Uniform distribution, Power distribution, and Upper Truncated Pareto distribution, all of which are frequently used in the literature.²² As emphasized in Section 2, I assume the upper bound of productivity distribution is less than infinity. I did not consider Pareto distribution in the theoretical or quantitative parts of our paper. On the one hand, the boundedness of $\bar{\varphi}$ is of theoretical importance. When the credit market is complete, *i.e.*, $\lambda \rightarrow \infty$, only the most productive entrepreneurs would take over

²²As shown in Appendix D, the above assumption is equivalent to assuming, for all $\tilde{\varphi} \in (\underline{\varphi}, \bar{\varphi})$, we have

$$\mathbb{E}_F \left[\left(\frac{\varphi}{\bar{\varphi}} \right)^{\frac{1}{\gamma}} \mid \varphi \in (\tilde{\varphi}, \bar{\varphi}) \right] \cdot \left\{ 1 - \left(\frac{1}{\gamma} \right) \cdot \left[\frac{1 - F(\tilde{\varphi})}{\tilde{\varphi} \cdot f(\tilde{\varphi})} \right] \right\} \leq 1.$$

the production. Models with a Pareto distribution would not be well defined in the extreme scenario, as emphasized by Moll (2012) and Wang and Wen (2013), who address heterogeneity in productivity and investment efficiency, respectively, with an incomplete financial market. On the other hand, our key channel through which credit imperfections affect unemployment would heavily depend on the above assumption. However $\Upsilon(\tilde{\varphi})$ would be purely constant if I adopt a Pareto distribution, and thus the transmission mechanism would be shut down in equilibrium. Therefore, I instead use a Power distribution with a normalized support $[0, 1]$ in the coming quantitative analysis.²³

Following the literature on business cycle accounting, such as Chari, Kehoe and McGrattan (2007), I characterize allocation and wedges of the decentralized economy in general equilibrium as below.

Proposition 2. (Wedges in General Equilibrium) Given the aggregate state variable X ,

1. the cut-off point $\hat{\varphi}$ increases with λ such that $\lim_{\lambda \rightarrow 1} \hat{\varphi} = \underline{\varphi}$ and $\lim_{\lambda \rightarrow \infty} \hat{\varphi} = \bar{\varphi}$.
2. the aggregate output and the total matched workers are

$$\begin{aligned} Y &= (1 - \tau_y) \cdot Y^* = (1 - \tau_y) \cdot \bar{\varphi} \cdot m(K, L) \\ N &= (1 - \tau_n) \cdot N^* = (1 - \tau_n) \cdot m(K, L) \end{aligned}$$

where

$$\begin{aligned} 1 - \tau_y &= \Lambda(\lambda) \equiv \mathbb{E}_F \left[\left(\frac{\varphi}{\bar{\varphi}} \right)^{\frac{1}{\gamma}} \mid \varphi \in [\hat{\varphi}, \bar{\varphi}] \right]^{\gamma} \in (0, 1) \\ 1 - \tau_n &= \Omega(\lambda) \equiv \frac{\mathbb{E}_F \left(\varphi^{\frac{1-\gamma}{\gamma}} \mid \varphi \in [\hat{\varphi}, \bar{\varphi}] \right)}{\left[\mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\hat{\varphi}, \bar{\varphi}] \right) \right]^{1-\gamma}} \in (0, 1). \end{aligned}$$

both of which increases with λ , and $\lim_{\lambda \rightarrow \infty} \tau_y = \lim_{\lambda \rightarrow \infty} \tau_n = 0$.

²³Uniform distribution is a special case of Power distribution. I use uniform distribution as an example in our theoretical analysis since it is a perfect candidate to exercise mean preserving spread. I then calibrate the parameters of Power distribution in the quantitative part. I also tried the Upper Truncated Pareto distribution.

3. the average labor productivity, $ALP \equiv \frac{Y}{N}$, and unemployment, $u \equiv 1 - \frac{N}{L}$, is²⁴

$$\begin{aligned} ALP &= (1 - \tau_{alp}) \cdot ALP^* = \mathbb{E}_{F_M}(\varphi) \\ u &\equiv (1 + \tau_u) \cdot u^* = u^* + \tau_n \cdot (1 - u^*) \end{aligned} \quad (1.16)$$

where

$$\begin{aligned} 1 - \tau_{alp} &= \frac{1 - \tau_y}{1 - \tau_n} = \Upsilon(\lambda) \equiv \frac{\mathbb{E}_{F^i} \left[\varphi_i^{\frac{1}{\gamma}} \mid \varphi_i \in [\widehat{\varphi}_i, \overline{\varphi}_i] \right]}{\mathbb{E}_{F^i} \left[\left(\frac{\overline{\varphi}_i}{\varphi_i} \right) \cdot \varphi_i^{\frac{1}{\gamma}} \mid \varphi_i \in [\widehat{\varphi}_i, \overline{\varphi}_i] \right]} \in (0, 1) \\ 1 + \tau_u &= 1 + \tau_n \cdot \left(\frac{1 - u^*}{u^*} \right) \in (1, \infty). \end{aligned}$$

4. the wedge to the expected labor revenue is zero, *i.e.*, $W = \frac{\partial Y}{\partial L}$ while the wedge to the interest rate is

$$r = (1 - \tau_r) \cdot \left(\frac{\partial Y}{\partial K} \right)$$

where $1 - \tau_r \equiv \frac{1}{\mathbb{E}_{F^i} \left[(\varphi_i / \widehat{\varphi}_i)^{\frac{1}{\gamma}} \mid \varphi_i \in [\widehat{\varphi}_i, \overline{\varphi}_i] \right]}$, which increases with λ , and $\lim_{\lambda \rightarrow \infty} \tau_r = 0$.

5. the equilibrium labor supply and the corresponding wage offer in sub market φ is

$$\begin{aligned} \frac{l(\varphi)}{L} &= \left[\frac{\varphi}{\Lambda(\lambda)} \right]^{\frac{1}{\gamma}} \cdot \left[\frac{v(\varphi)}{Kf(\varphi)} \right] \\ w(\varphi) &= (1 - \gamma) \cdot \varphi \cdot \mathbf{1}_{\{\varphi \geq \widehat{\varphi}(\lambda)\}}, \end{aligned}$$

and the cumulative distribution is $F_w(\omega) \equiv Pr\{w \leq \omega\} = F_M\left(\frac{\omega}{1-\gamma}\right)$, where $F_M(\cdot)$ denotes the equilibrium productivity distribution of the capital they are matched with labor.

First, both ALP and N increase with λ . Therefore, credit imperfections affect the output not only through lowering capital misallocation, *i.e.*, the decrease of ALP, but also by alleviating labor misallocation, *i.e.*, the increase of employment. The former

²⁴I use $\lambda = \infty$ as the limit case for our theoretical analysis. If I use some $\bar{\lambda} < \infty$ instead as the limit scenario, then the formula between u and u^* is adjusted as $u = \bar{u} + \left[1 - \frac{\Omega(\lambda)}{\Omega(\bar{\lambda})} \right] \cdot (1 - \bar{u})$, where $\bar{u} \equiv 1 - \Omega(\bar{\lambda}) \cdot m\left(\frac{K}{L}, 1\right)$.

and latter denote the intensive and extensive margins, respectively. Therefore our model offers a new channel through which a credit crunch generates an amplification effect on output. I further illustrate this result in the quantitative exercise in Section 4.

Second, given $\varphi \geq \widehat{\varphi}$, both $v(\varphi)$ and $l(\varphi)$ increases with λ . However, as shown in the above proposition, $l(\varphi)$ does not increase as much as $v(\varphi)$ does. Therefore, the market tightness $\theta(\varphi) \equiv l(\varphi)/v(\varphi)$ and the associated job-filling rate $q(\varphi)$ decreases with λ in general equilibrium. That is, as more capital is concentrated at the top end, the market tightness tends to be less favorable to firms.

Third, Proposition 2 provides a micro-foundation for the Cobb-Douglas aggregation. In turn, equilibrium TFP is defined as

$$TFP(\lambda, \eta, z) \equiv \frac{Y}{K^\gamma L^{1-\gamma}} = [ALP(\lambda, z)] \cdot [\Omega(\lambda) \cdot \eta], \quad (1.17)$$

which is determined by aggregate productivity and frictions to credit and labor markets. Therefore, credit imperfections affect equilibrium TFP at intensive margin (capital misallocation) as well as extensive margin (employment). I can also characterize TFP wedge as $TFP \equiv (1 - \tau_{tfp}) \cdot TFP^*$ and thus $\tau_{tfp} = \tau_y$. Moreover, following Lagos (2006), I can alternatively use the finally matched capital and labor, *i.e.*, $L_M = K_M = N$ to measure equilibrium TFP. Then I have $\widetilde{TFP} \equiv \frac{Y}{K_M^\gamma L_M^{1-\gamma}} = \frac{Y}{N} = ALP$, which is affected by both z and λ . However, it is independent of η since matching efficiency only affects matched capital and labor.

I have characterized at the end of Section 2 the intertemporal decision of individual entrepreneurs. I close this part by characterizing the aggregate transition dynamics.

Corollary 3. (Aggregate Transition Dynamics)

$$\begin{aligned} K_t &= \beta \cdot [\gamma \cdot Y_t + (1 - \delta) \cdot K_t]. \\ G_{t+1}(a) &= \int G_t \left(\frac{a}{\beta \cdot \Psi_t(\varphi)} \right) \cdot dF_t(\varphi). \end{aligned}$$

The evolution of aggregate capital stock behaves like a Solow model in which output is subject to a tax rate $(1 - \gamma)$ and the saving rate is constant. On the one hand, the Cobb-Douglas matching function in all sub-labor markets suggests a fixed split of output between entrepreneurs and workers. Since I assume workers cannot have access to the credit market, only entrepreneurs make intertemporal decisions.

On the other hand, I use log-utility, which exactly cancels income and substitution effects and implies a fixed saving rate.

1.3.2 The Unemployment Effect of Credit Imperfections

The key theoretical contribution of this paper is to show that a credit crunch, *i.e.*, a decrease in λ , lowers aggregate matching efficiency. I use this section to present the details of this new transmission mechanism. As shown in the proof of Proposition 2, equilibrium employment can be formulated as

$$N = \mathbb{E}_F [q(\varphi) | \varphi \geq \hat{\varphi}] \cdot K, \quad (1.18)$$

where K denotes the aggregate capital supply and $q(\varphi)$ the job-filling rate in sub-labor market φ . In turn I obtain the employment effect of credit imperfections as below.

$$\frac{\partial N}{\partial \lambda} = \left\{ \left(\frac{\partial \mathbb{E}_F [q(\varphi) | \varphi \geq \hat{\varphi}]}{\partial \hat{\varphi}} \right) \cdot \left(\frac{\partial \hat{\varphi}}{\partial \lambda} \right) + \mathbb{E}_F \left[\frac{\partial q(\varphi)}{\partial \lambda} | \varphi \geq \hat{\varphi} \right] \right\} \cdot K \geq 0 \quad (1.19)$$

First, the increase of λ drives up the interest rate r and thus the cut-off value $\hat{\varphi}$. Then more capital is redistributed from low-productivity to high-productivity entrepreneurs. As proved in Section 2.2, the entrepreneur's job-filling rate $q(\varphi)$ increases with φ . Therefore, the direct effect, which is shown in the first item of the right hand of Equation (1.19), is that the employment increases. I call it the *selection effect*. However, holding everything else unchanged, when more capital is concentrated in the hands of high-productivity entrepreneurs, the job-filling rate of all active entrepreneurs tends to decrease. Although labor supply responds to the increase of λ , the concavity of the matching function suggests $l(\varphi)$ does not change as much as $v(\varphi)$ and thus $q(\varphi)$ decreases with λ in general equilibrium. This can be verified from the above proposition. This indirect general equilibrium effect is labeled as the *congestion effect*, which is shown in the second item of the right hand of Equation (1.19). As proved in Appendix D, the selection effect dominates the congestion effect under Assumption 1.

As suggested by Equation (1.16), productivity heterogeneity with an incomplete credit market does matter for matching efficiency in the labor market. Such an effect cannot be obtained in a standard framework with a representative firm and

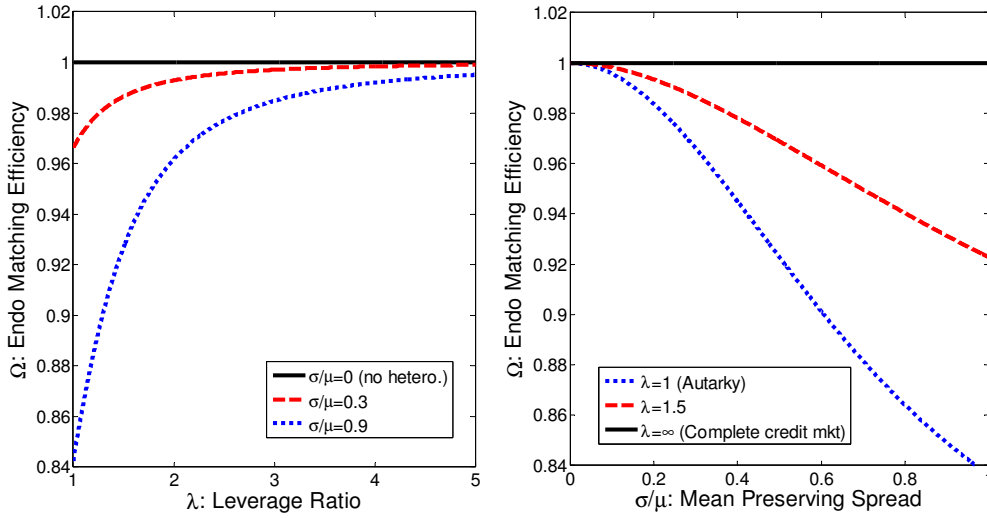


Figure 1.7: **Left Panel:** (Ω, λ) ; **Right Panel:** $(\Omega, \frac{\sigma}{\mu})$; $\varphi \stackrel{U}{\sim} [\mu - \sigma, \mu + \sigma]$.

worker. For example, the seminal work by Wasmer and Weil (2004) introduces credit frictions into an otherwise standard Diamond-Mortensen-Pissarides model. They model credit frictions with a matching function between a representative firm and bank. When credit frictions worsen, which could be driven by the decrease of matching efficiency between firms and banks, this affects equilibrium unemployment in the steady state. However, unlike our heterogeneity model, the Beveridge curve in their work does not move with such kind of disruption in the credit market.

Finally, endogenous matching efficiency contributed by credit imperfections, Ω , is affected not only by λ , but also by the productivity distribution. Given any distribution $F(\cdot)$, I have shown Ω increases with λ . I close this section by addressing the implications of an MPS (mean preserving spread) of $F(\cdot)$ for Ω . The general discussion is beyond the scope of this paper. I instead use a special case to illustrate the idea by assuming $F(\cdot)$ is a Uniform distribution with support $[\mu - \sigma, \mu + \sigma]$ and $\sigma \in [0, \mu]$. I use a uniform distribution since it is a perfect candidate to perform MPS. More specifically, given any λ , I can check the effect of $\frac{\sigma}{\mu}$ on Ω . The right panel of Figure (1.7) implies an MPS increases unemployment. Our exercise with MPS is related to the recent literature on the relationship between adverse selection and output fluctuation; see Kurlat (2012) and Bigio (2013), among others. I show that an MPS depresses the output. There are mainly two key differences. First, information asymmetry is indispensable in their work while I perform the MPS using complete information. Second, they assume a frictionless labor market while I assume labor search frictions and an MPS drives up unemployment.

1.3.3 Unemployment Decomposition

Motivated by the channel through which credit imperfections affect the labor market, I make a theoretical decomposition for unemployment in this section. In particular, I explore how much credit imperfections and the classic labor search frictions add to unemployment.

Steady State

Using Corollary 3 reaches steady-state unemployment as below.

$$u_{ss} = 1 - [\Omega(\lambda_{ss}) \cdot \eta_{ss}]^{\frac{1}{1-\gamma}} \cdot \left[\frac{\gamma \cdot ALP(\lambda_{ss}, z_{ss})}{1/\beta - 1 + \delta} \right]^{\frac{\gamma}{1-\gamma}}, \quad (1.20)$$

As indicated by Equation (1.20), the credit friction λ plays two roles in determining unemployment in the steady state. On the one hand, the increase of λ contributes to a higher TFP, which in turn suggests a higher capital stock in the steady state. Therefore, unemployment tends to decrease. On the other hand, given any level of capital stock, the endogenous matching efficiency would also increase with λ and thus lower unemployment. In the end, I reach the general equilibrium effect of credit imperfections on unemployment in the steady state as below.

$$(u_{ss} - u_{ss}^*) = (u_{ss} - \tilde{u}) + (\tilde{u} - u_{ss}^*),$$

where u_{ss} and u_{ss}^* denote, respectively, the steady state unemployment with a steady state λ and with a “high enough” λ . The difference between u_{ss} and u_{ss}^* is defined as unemployment contributed by credit imperfections in the steady state. Furthermore, \tilde{u} is denoted as unemployment implied by a higher λ , but the matching efficiency is held constant. That is, \tilde{u} is the steady state unemployment with a higher capital stock implied by an improvement of capital reallocation, but the efficiency of labor reallocation is held unchanged. I have formulated u_{ss} in Equation (1.20). In turn, u_{ss}^* and \tilde{u} are given as below.

$$\begin{aligned} u_{ss}^* &\equiv 1 - [\Omega(\lambda^*) \cdot \eta_{ss}]^{\frac{1}{1-\gamma}} \cdot \left[\frac{\gamma \cdot ALP(\lambda^*, z_{ss})}{1/\beta - 1 + \delta} \right]^{\frac{\gamma}{1-\gamma}} \\ \tilde{u} &\equiv 1 - [\Omega(\lambda_{ss}) \cdot \eta_{ss}]^{\frac{1}{1-\gamma}} \cdot \left[\frac{\gamma \cdot ALP(\lambda^*, z_{ss})}{1/\beta - 1 + \delta} \right]^{\frac{\gamma}{1-\gamma}}, \end{aligned}$$

where λ^* denotes a “high” financial development. I have two alternative candidates for λ^* , one is ∞ while the other one is $\max\{\lambda_t\}$. The former is mainly of theoretical interest. As proved in Proposition 2, endogenous matching efficiency by credit imperfections would converge to the maximum level when λ^* approaches

infinity. The latter is instead used for the quantitative analysis presented below.

Non-Steady State

In every period I have $u = 1 - \Omega(\lambda) \cdot m\left(\frac{K}{L}, 1\right)$. That is, the total matching efficiency is the product of that contributed by financial friction and that by labor search frictions, *i.e.*, $\hat{\eta} = \Omega(\lambda) \cdot \eta$. Then I have

$$u = u^{**} + \left(\lim_{\lambda \rightarrow \lambda^*} u - u^{**} \right) + \left(u - \lim_{\lambda \rightarrow \lambda^*} u \right) \equiv u^{**} + u^\eta + u^\lambda,$$

where $u^{**} = \max\left\{1 - \frac{K}{L}, 0\right\}$ and $\lim_{\lambda \rightarrow \lambda^*} u = 1 - \Omega(\lambda^*) \cdot m\left(\frac{K}{L}, 1\right)$ denote, respectively, the efficient unemployment and the unemployment without credit imperfections. First, data on $\frac{K}{L}$ suggest $u^{**} = 0$. Then I break down unemployment into two parts: one is due to the classic search friction while the other is due to credit imperfections. I denote them as u^η and u^λ , respectively. In turn, I define the explanatory power of credit imperfections on unemployment as $\chi \equiv \frac{u^\lambda}{u}$. Given K , aggregate productivity shock z does not directly affect unemployment since z has nothing to do with the equilibrium aggregate matching efficiency. Therefore, the decomposition exercise does not involve z . However, z exerts a dynamic effect on unemployment because aggregate productivity shock plays a role in equilibrium TFP, which in turn influences the speed of capital accumulation.

Finally, I get that $\frac{\partial \chi}{\partial \lambda} < 0$, $\frac{\partial \chi}{\partial \eta} > 0$, $\frac{\partial^2 \chi}{\partial \lambda \partial \eta} < 0$ and $\frac{\partial \chi}{\partial \lambda^*} > 0$. The increase of λ suggests an amelioration of capital misallocation, and thus the role of credit imperfections in explaining unemployment decreases. As a duality, I have $\frac{\partial(1-\chi)}{\partial \eta} < 0$, which immediately translates into $\frac{\partial \chi}{\partial \eta} > 0$. Furthermore, there exists an interaction effect. These properties turn out to be helpful in interpreting the results, mainly Figure (1.14), in Section 4.3. I illustrate the key results on χ in Figure (1.8).

1.3.4 The Relationship to A Model with Only Credit Frictions

I have finished the specification and characterization of our model with credit and labor-search frictions. Comparing the equilibrium of the decentralized economy with the benchmark with only search frictions delivers Proposition 2. I address the wedges to productivity, employment, average labor productivity, unemployment rate and factor prices there.

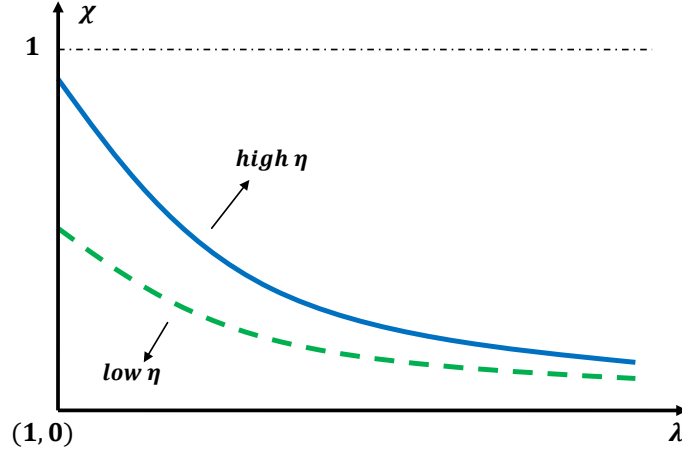


Figure 1.8: **Explanatory Power of Credit Imperfections for Unemployment**

I use this section to propose an alternative benchmark in which there is no labor search friction but just credit friction. More specifically, I compare our model with Moll (2013). I establish the connection as below.

Proposition 3. (Comparison with a Model with Only Credit Frictions)

The heterogeneous-entrepreneurs model with both search frictions in the labor market with matching function $m(l(\varphi), v(\varphi)) = \eta \cdot v(\varphi)^\gamma l(\varphi)^{1-\gamma}$, and credit frictions in the form of a collateral constraint $k \leq \lambda \cdot a$ delivers the same output aggregation and transition dynamics on $F(\varphi)$, $G(a)$ and K with the model with the following characteristics:

1. The production function by entrepreneur- (a, φ) is $y(\varphi, a) = \varphi \cdot m(k(\varphi, a), l(\varphi, a))$.
2. The labor market is frictionless in each period, *i.e.*, there exists a unique and publicly displayed wage w such that labor supply equals demand. Unemployment is then zero by definition.
3. The credit market is subject to a collateral constraint, *i.e.*, $k \leq \lambda \cdot a$.

Moreover, I use the model with only financial friction to recover unemployment in the model with dual frictions as $u = 1 - \frac{Y}{L \cdot \mathbb{E}_{F_M}(\varphi)}$.

The key message from this proposition is that, our model with two layers of frictions behaves as if there exist only credit imperfections, and the Leontief production

function is replaced by the Cobb-Douglas. On the other hand, I interpret the heterogeneous model with only credit frictions as a model with both credit and labor search frictions, and the Cobb-Douglas production function is decomposed into a Leontief production and an associated Cobb-Douglas matching function.

1.3.5 Job Destruction and Firm Growth

I have assumed throughout the paper that all matched relationships between firms and workers are terminated after production. This assumption greatly simplifies our analysis since entrepreneurs are heterogeneous in only two dimensions. The associated cost is that, the ratio of job destruction to total employment is 100% at the end of each period. To partially fix this problem, I redefine job destruction in terms of net flow.

$$N_{t+1} = N_t - JD_{t+1} + JC_{t+1}, \quad (1.21)$$

where

$$\begin{aligned} N_t &\equiv \int \int \tilde{l}_t h_t(\varphi, a) d\varphi da = \Omega(\lambda_t) \cdot m_t(K_t, L_t) \\ JD_{t+1} &\equiv \int \int \max\{\tilde{l}_t - \tilde{l}_{t+1}, 0\} h_t(\varphi, a) d\varphi da \\ JC_{t+1} &\equiv \int \int \max\{\tilde{l}_{t+1} - \tilde{l}_t, 0\} h_t(\varphi, a) d\varphi da \end{aligned}$$

and \tilde{l}_t denotes the finally matched workers, and $h(\varphi, a)$ the joint distribution of productivity and net worth. Given N_t, N_{t+1} and JD_{t+1} , I can then calculate JC_{t+1} from Equation (1.21). Moreover, in each period, given the aggregate state variable X_t , I can pin down N_t . Therefore, it remains for us to characterize JD_{t+1} . I summarize the key results below.

Corollary 4. (Job Creation and Destruction) In each period, job destruction can be formulated as

$$JD_{t+1} = \left[\int \max(\Delta_{t,t+1}(\varphi_t), 0) \cdot dF(\varphi_t) \right] \cdot K_{t+1}.$$

where

$$\Delta_{t,t+1}(\varphi_t) \equiv \lambda_t \cdot \mathbf{1}_{\{\varphi_t \geq \hat{\varphi}_t\}} q_t(\varphi_t) - \lambda_{t+1} \beta \Psi_t(\varphi_t) \cdot \left[\rho \cdot \mathbf{1}_{\{\varphi_t \geq \hat{\varphi}_{t+1}\}} q_{t+1}(\varphi_t) + (1 - \rho) \cdot \int_{\Phi} \mathbf{1}_{\{\varphi \geq \hat{\varphi}_{t+1}\}} q_{t+1}(\varphi) \cdot dF(\varphi) \right].$$

Using the corollary immediately suggests that job creation and job destruction in the steady state is

$$JD = JC = \left[\int_{\widehat{\varphi}}^{\overline{\varphi}} \max \left\{ 1 - \beta \Psi(\varphi) \cdot \left[\rho \cdot q(\varphi) + (1 - \rho) \cdot \left(\frac{N}{\lambda K} \right) \right], 0 \right\} dF(\varphi) \right] \cdot \lambda K.$$

where K and N in the steady state are

$$\begin{aligned} K &= \left[\frac{\gamma \cdot TFP(\lambda, \eta, z)}{1/\beta - (1 - \delta)} \right]^{\frac{1}{1-\gamma}} \cdot L \\ N &= \Omega(\lambda) \cdot m(K, L). \end{aligned}$$

I close the theoretical section with a discussion on the implications of this model for firm-level growth.

Corollary 5. (Firm Size and Growth Rate)

1. The firm size of entrepreneur- (a, φ) , measured by capital holding k and employee numbers n , are

$$\begin{aligned} k(a, \varphi) &= \lambda a \cdot \mathbf{1}_{\{\varphi \geq \widehat{\varphi}\}} \\ n(a, \varphi) &= \lambda q(\varphi) a \cdot \mathbf{1}_{\{\varphi \geq \widehat{\varphi}\}}. \end{aligned}$$

2. The growth rate of capital and employment is,

$$\begin{aligned} \mathbb{E} \left[\frac{k_{t+1}}{k_t} | (k_t, \varphi_t; X_t) \right] &= \beta \cdot \left(\frac{\Psi_t(\varphi_t)}{\lambda_t} \right) \cdot \mathbb{E} \left\{ \left[\rho \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_{t+1}\}} + (1 - \rho) \cdot (1 - F(\widehat{\varphi}_{t+1})) \right] \cdot \lambda_{t+1} | (\varphi_t, X_t) \right\} \\ \mathbb{E} \left[\frac{n_{t+1}}{n_t} | (n_t, \varphi_t; X_t) \right] &= \beta \cdot \Psi_t(\varphi_t) \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left[\rho \cdot \left(\frac{q_{t+1}(\varphi_t)}{q_t(\varphi_t)} \right) + (1 - \rho) \cdot \left(\frac{\int_{\widehat{\varphi}_{t+1}}^{\overline{\varphi}} q(\varphi_{t+1}) dF(\varphi_{t+1})}{q_t(\varphi_t)} \right) \right]. \end{aligned}$$

Our model predicts that the growth rate has nothing to do with capital or employment itself. This is purely because of the linearity of the policy function of individual entrepreneurs. Moreover, the heterogeneous growth rate connects our paper with the empirical and theoretical works on the volatility of firm growth rate, such as that by Wang and Wen (2013).

1.4 Quantitative Analysis

In this section I confront the model with data by quantifying the unemployment effect of capital misallocation due to credit imperfections. I calibrate the model on US economy and then recover the realization of three pieces of aggregate shocks. Then I estimate the stochastic process of these three shocks and do the impulse response exercise. Furthermore, I decompose unemployment into two parts, one which is due to credit imperfections and the other which is due to the classic labor search frictions. Unemployment decomposition is considered not only for the business cycles between 1951Q4 and 2011Q4, but also for the recent financial crisis. Finally, I discuss which shocks are most essential in terms of their aggregate and disaggregate implications.

1.4.1 Calibration

Data are of quarterly frequency. Details on the criterion of data use, data description and calculation are documented in Appendix A. The date horizon ranges between 1951Q4 to 2011Q4.²⁵

Calibration

Assume the individual productivity component follows a Power distribution, *i.e.*, $F(\varphi) = \varphi^{\frac{1}{\varepsilon}}$ with the support $[\varphi, \bar{\varphi}] = [0, 1]$ and $\varepsilon > 0$. Notice that the lower bound is truncated at zero since productivity is non-negative by definition. Besides, I normalize the upper bound by one.²⁶ The empirical literature reveals the probability density of the function of the productivity distribution decreases at the right tail. Therefore, I should have $\frac{1}{\varepsilon} < 1$. The calibration in the following Table (1.1) confirms the empirical regularity.

²⁵The data start with 1951Q4 since it is the earliest date in the Flow of Funds Account such that the data on external funding on non-financial assets are available. The date ends with 2011Q4 since this is the last date in which the quarterly data on non-financial private investment are obtainable. Although the annual data on capital are available until 2012, I have to use both the annual data on capital and the quarterly data on investment to recover the quarterly data on capital.

²⁶In general, I can specify the distribution as $F(\varphi) = (\varphi/\bar{\varphi})^{\frac{1}{\varepsilon}}$. In this case, I can show that $\hat{\varphi}$ is homogeneous of degree one with respect to $\bar{\varphi}$. Therefore, without loss of generality, I can normalize $\bar{\varphi} \equiv 1$.

Parameter	Note
$\beta = 0.99$	discount factor
$\delta = 0.025$	depreciation rate
$[\underline{\varphi}, \bar{\varphi}] = [0, 1]$	support of the Power distribution
$\gamma = 0.28$	matching elasticity <i>and</i> revenue share of capital
$\eta = 0.61$	matching efficiency of labor market
$\lambda = 1.4$	collateral constraint
$\rho = 0.91$	persistence of individual productivity
$z = 6.4$	aggregate productivity
$\varepsilon = 1.68$	parameter in the Power distribution

Table 1.1: **Calibration**

I target the annual interest rate as 4% by setting the quarterly discount factor as $\beta = 0.99$. As suggested in Bigio (2013), the combination of $\beta = 0.99$ and log utility function are quantitatively similar to that of $\beta = 0.97$ and the coefficient of risk aversion being 2. Therefore, I use $\beta = 0.97$ throughout the quantitative analysis. As standard in the literature, I set the depreciation rate as 2.5%. TFP is defined as $Y / (K^\gamma \cdot L^{1-\gamma})$. In turn, TFP_{ss} is a function of γ . Moreover, as shown in Section 3.3, the capital per capita in the steady state is $\left(\frac{\gamma \cdot TFP_{ss}}{1/\beta - 1 + \delta} \right)^{\frac{1}{1-\gamma}}$, which is also related to γ . I therefore recover $\gamma = 0.281$ from the average capital per capita. On the one hand, γ denotes the elasticity of the matching function, which was estimated between 0.28 by Shimer (2005) and 0.5 by Petrongolo and Pissarides (2001). On the other hand, γ stands for the revenue share of capital, which was set as 0.25 by Cui (2013), 0.28 by Gomme and Rupert (2007), and 0.33 by Buera, Fattal-Jaef and Shin (2013), among others. Our estimation $\gamma = 0.28$ falls into the intervals of values used by previous research.

Using data from JOLTS, I construct quarterly data on job-filling rates and market tightness. I follow Michaillat (2012) to use OLS to estimate η . I get the steady state λ by averaging the ratio of external funding over non-financial assets and using the relationship that $\frac{D}{K} = 1 - \frac{1}{\lambda}$. In turn, I obtain ρ , the persistence coefficient of individual productivity, by using 10%, the annual exit rate of establishment as below. On the one hand, by definition I have $1 - \{\rho + (1 - \rho) \cdot [1 - F(\hat{\varphi})]\}^4 = 0.1$. On the other hand, the clearing condition in the credit market suggests $\lambda \cdot [1 - F(\hat{\varphi})] = 1$. Therefore, I get ρ as $\rho = \left(0.9^{\frac{1}{4}} - \frac{1}{\lambda} \right) / \left(1 - \frac{1}{\lambda} \right) = 0.91$. Alternatively, I can use $\frac{JD}{N}$, the ratio of job destruction to total employment, to pin down ρ . The data on JD and N are available from the Bureau of Labor Statistics, which suggest $\frac{JD}{N} = 1.5\%$

on average. In turn, I have $\rho = 0.99$.²⁷

When it comes to the estimation on the distribution parameter ε , I can first construct the series of average labor productivity (ALP) by dividing the output by the employment. Additionally, using the theoretical results in Section 3 suggests that $\Omega = TFP / (ALP \cdot \eta)$, which delivers the steady state of matching efficiency by credit imperfections. Meanwhile, notice that Ω is a function of the distribution parameter ε and that of the steady-state λ . Thus I recover $\varepsilon = 1.684$, which suggests that $1/\varepsilon < 1$. Therefore, the pdf decreases with productivity and thus is in line with empirical regularity. Finally, I reach the steady-state value of z from TFP .

Backing Out Shocks

There are three aggregate shocks in our model, $\{\lambda_t, \eta_t, z_t\}$, which are not directly observable from the data. I recover these shocks from certain observable time series by following Michaillat (2012). On the one hand, I have data on i) $\frac{D_t}{K_t}$, the external funding over non-financial assets, ii) ALP_t , the average labor productivity, and iii) TFP_t . On the other hand, all three variables of interest are functions of $\{\lambda_t, \eta_t, z_t\}$ in our model, *i.e.*,

$$\begin{aligned}\frac{D}{K} &\equiv \frac{\int_0^\infty \int_{\underline{\varphi}}^{\bar{\varphi}} \max\{k(\varphi, a) - a, 0\} h(\varphi, a) d\varphi da}{K} = \left(\frac{D}{K}\right)(z, \lambda, \eta) \\ ALP &\equiv \frac{Y}{N} = ALP(z, \lambda, \eta) \\ TFP &\equiv \frac{Y}{K^\gamma L^{1-\gamma}} = TFP(z, \lambda, \eta).\end{aligned}$$

Furthermore, I can verify the diagonal property holds such that i) $\frac{D}{K} = \left(\frac{D}{K}\right)(\lambda)$, ii) $ALP = ALP(z, \lambda)$, and iii) $TFP = TFP(\eta, z, \lambda)$.²⁸ Therefore, in each period I can first use $\left(\frac{D}{K}\right)_t$ to recover λ_t . Then z_t could be inferred by jointly using ALP_t and the already recovered λ_t . Finally, I use TFP_t alongside the pairwise estimated value on (λ_t, z_t) to retrieve η_t . In turn, I obtain their corresponding HP filter in Figure (1.9).

²⁷Since the entrepreneur may lose her productivity with probability $(1 - \rho)$ and then may redraw a very low productivity, she may stop hiring then. Consequently, $\frac{D}{K}$ is a function of ρ in steady state.

²⁸Notice that the ratio of external funding to non-financial asset can be simplified as $\frac{D}{K} = 1 - \frac{1}{\lambda}$.

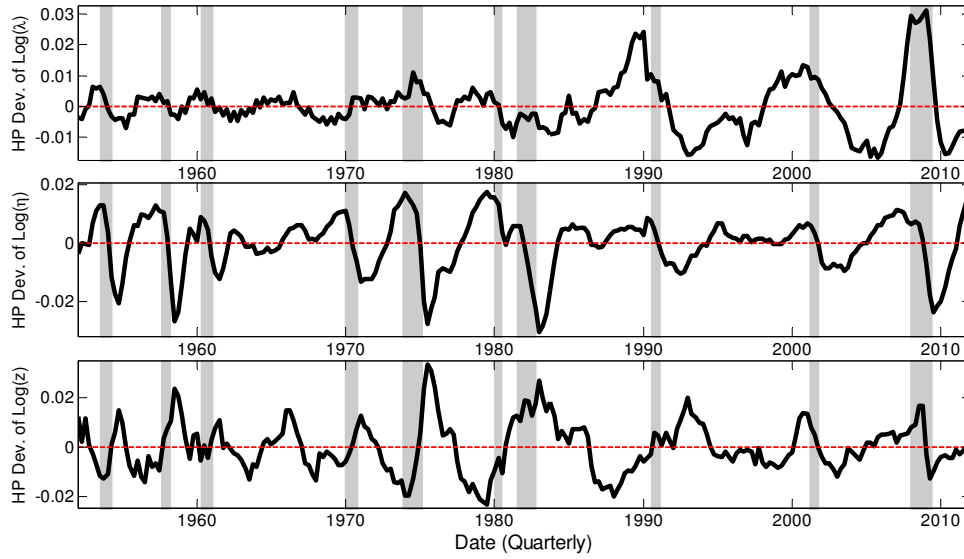


Figure 1.9: **HP Deviation of Three Shocks**

All these shocks are procyclical in general.²⁹ First, all three aggregate shocks were significantly negative in the recent financial crisis, especially for λ , the shock to the credit market. Moreover, both λ and η decreases in recessions over the cycles. Notice that I have adopted in a reduced-form way to model the credit and labor search frictions. On the one hand, as discussed in Section 2.3, the decrease of λ in recessions may originate from a worsening condition in adverse selection, moral hazard, costly state verification or limited enforcement. On the other hand, the negative shock to η may be due to the decrease of aggregate matching efficiency, which is in turn caused by some sector-specific shocks, as shown by Mehrotra and Sergeyev (2012). Alternatively, the decrease in η may stem from the job polarization proposed by Jaimovich and Siu (2013). The results are mixed when it comes to the cyclicity of z , aggregate productivity shock. The shocks to z were also negative in the past three recessions, but just opposite for the previous recessions. Our quantitative exercise is in line with their findings. It is worth noting that, although aggregate productivity z increased in some recessions, it is not necessarily true that equilibrium TFP also increased correspondingly. As shown in Figure (1.10), TFP is procyclical with a correlation of 0.91 with the output.³⁰

²⁹The correlation between output and (λ, η, z) , after HP filtering, is 0.44, 0.64 and 0.21, respectively. Besides, after HP filtering, $\text{corr}(\lambda, \eta) = 0.21$, $\text{corr}(\lambda, z) = -0.16$ and $\text{corr}(\eta, z) = -0.52$.

³⁰There is seemingly no consensus on the movement of TFP for the recent recession. Petrosky-Nadeau (2012) proposes a model to explain why TFP increased in this recession. However, as shown in our calculation, TFP, along with the output, suffered a significant decrease in the past financial recession. This may be due to different measurement methods.

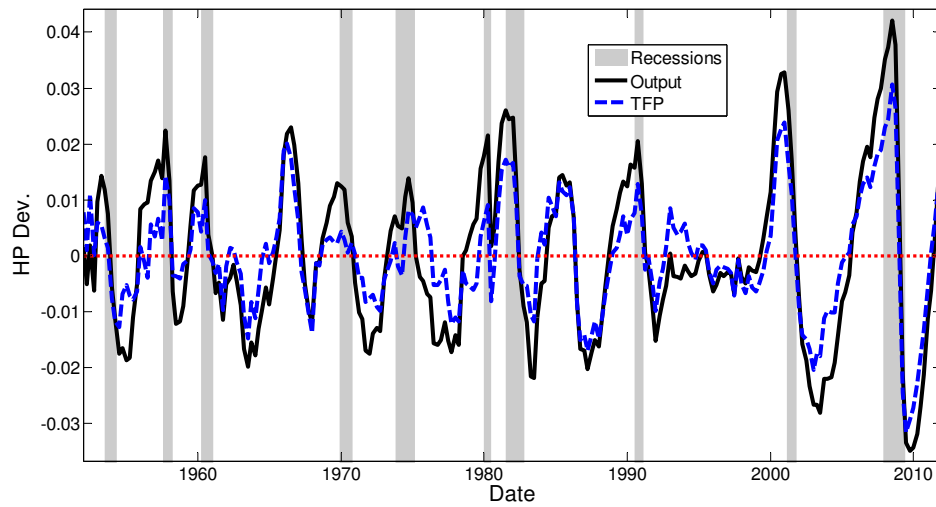


Figure 1.10: **HP Deviation of TFP**

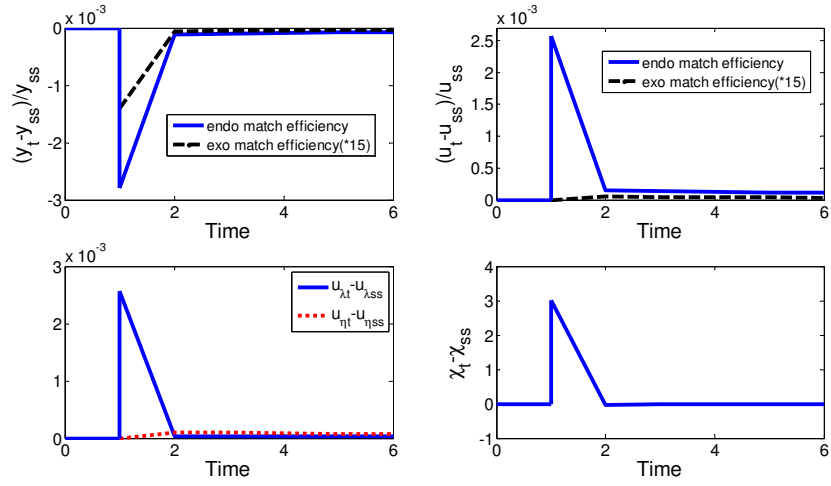
1.4.2 Impulse Response Exercise and Jobless Recovery

Now I investigate the implications of the aggregate shocks for output and unemployment. I also address their effects on unemployment decomposition.

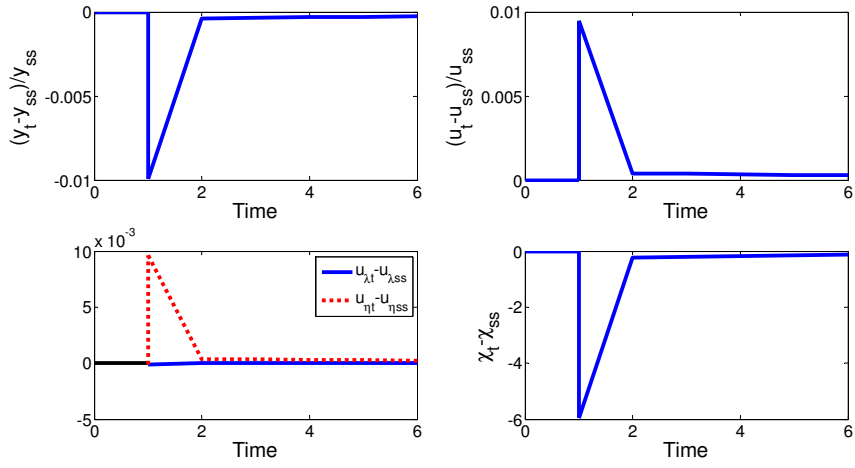
Impulse Response without Correlation or Persistence

I assume these three shocks decrease 1%, but with no persistence or correlation. I summarize the impulse response of output and unemployment in the first row of each panel in Figure (1.11). On the one hand, the path of output driven by different shocks shares a similar pattern. On the other hand, the implications of the shocks are different when it comes to unemployment. The credit and the labor market shocks exerts a large and immediate response for unemployment. On the contrary, aggregate productivity affects unemployment one period later and generates a relatively slow recovery.

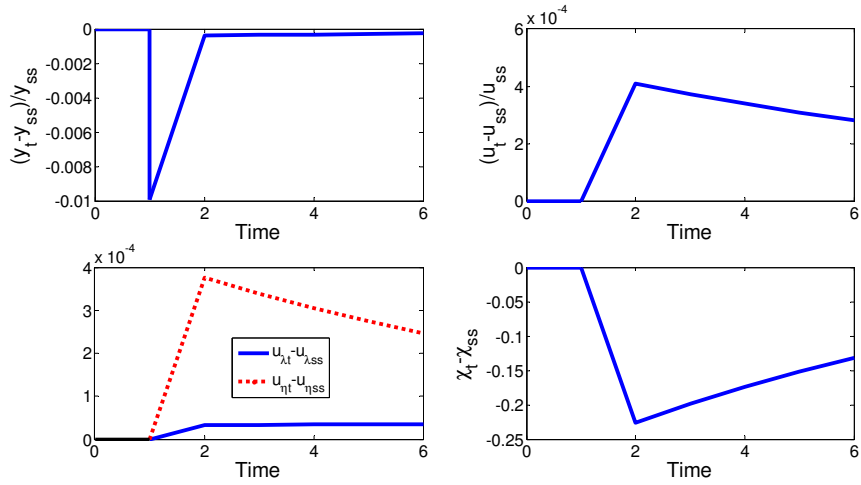
Since I mainly focus on the connection between credit and labor markets, I devote more analysis in this line. As illustrated in Section 3.2, credit imperfections lowers aggregate matching efficiency. In the upper panel of Figure (1.11), I compare the



(a) λ -shock



(b) η -shock



(c) z -shock

Figure 1.11: Impulse Response of Shocks (1%) without Correlation or Persistence

effect of credit crunches in two scenarios, one with endogenous matching efficiency and the other with exogenous matching efficiency. As shown in the upper panel, endogenous matching efficiency due to credit imperfections amplifies the effect of a credit crunch for output as well as for unemployment. Moreover, the amplification is quantitatively important.

Now I address the implications of the shocks for unemployment decomposition (u^λ, u^η) in the second row of each panel in Figure (1.11).³¹ First, although credit and labor shocks have similarity in their effect on output and unemployment, their predictions on χ , the explanatory power of credit imperfections on unemployment, are opposite. These results corroborate the theoretical predictions in Section 3.3, *i.e.*, $\frac{\partial \chi}{\partial \lambda} < 0$ and $\frac{\partial \chi}{\partial \eta} > 0$. On the one hand, a negative credit shock, *i.e.*, a credit crunch, boosts the importance of credit imperfections in explaining the associated increasing unemployment. On the other hand, a negative labor shock, *i.e.*, the matching efficiency decreases, puts more emphasis on the responsibility of the labor market itself in worsening unemployment. Moreover, these two shocks exert a qualitatively asymmetric effect on (u^λ, u^η) . The former increases both unemployment compositions while the latter increases u^η while decreases with u^λ . Finally, as predicted by Section 3.4, aggregate productivity shock does not directly affect unemployment in the current period. It works through capital accumulation and thus affects unemployment in the next period. The decrease of capital stock in turn attenuates the importance of credit imperfections while increases the importance of labor search frictions in explaining the rising unemployment. However, relative to the previously two shocks, aggregate productivity shock does not affect χ significantly when it comes to the quantitative concern.

VAR Estimation and Jobless Recovery

I have so far demonstrated the effects of the various shocks. Using the HP deviations of these three shocks delivers the estimation of the VAR process on (λ_t, η_t, z_t) . I document the estimated coefficient and variance matrix as below,

³¹The theory formulated in Section 3.3 proposes an unemployment decomposition, *i.e.*, $u = u^\eta + u^\lambda$, where $u^\eta \equiv \lim_{\lambda \rightarrow \max\{\lambda_t\}} u$ and u^λ denotes unemployment contributed by the labor search frictions and credit imperfections respectively. In turn, I define the explanatory power by credit imperfections to unemployment as $\chi \equiv u^\lambda/u$.

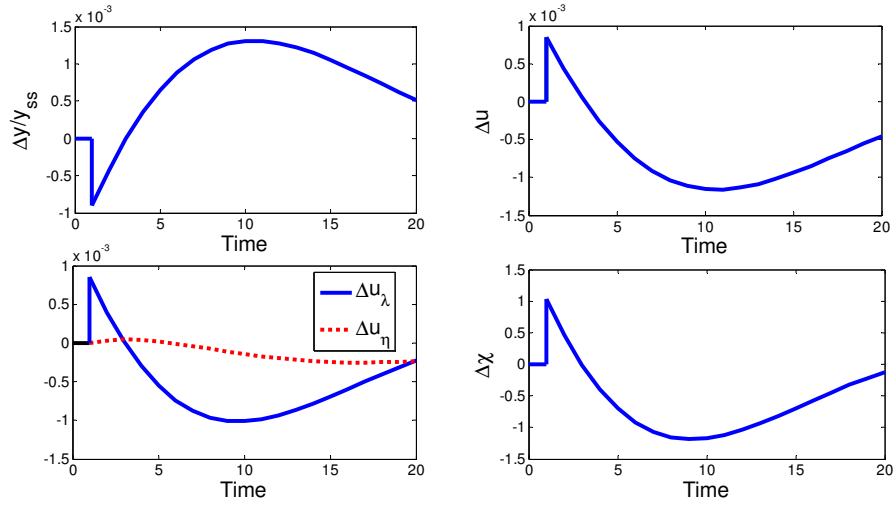
$$\begin{pmatrix} \beta_{\lambda\lambda} & \beta_{\lambda\eta} & \beta_{\lambda z} \\ \beta_{\eta\lambda} & \beta_{\eta\eta} & \beta_{\eta z} \\ \beta_{z\lambda} & \beta_{z\eta} & \beta_{zz} \end{pmatrix} = \begin{pmatrix} 0.894 & 0.147 & 0.054 \\ -0.120 & 0.905 & -0.004 \\ -0.009 & -0.003 & 0.865 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sigma_\lambda^2 & \rho_{\lambda z} \cdot \sigma_\lambda \sigma_z & \rho_{\lambda\eta} \cdot \sigma_\lambda \sigma_\eta \\ \rho_{\lambda z} \cdot \sigma_\lambda \sigma_z & \sigma_z^2 & \rho_{z\eta} \cdot \sigma_z \sigma_\eta \\ \rho_{\lambda\eta} \cdot \sigma_\lambda \sigma_\eta & \rho_{z\eta} \cdot \sigma_z \sigma_\eta & \sigma_\eta^2 \end{pmatrix},$$

where β_{ij} denotes the effect of shock- j on shock- i , and

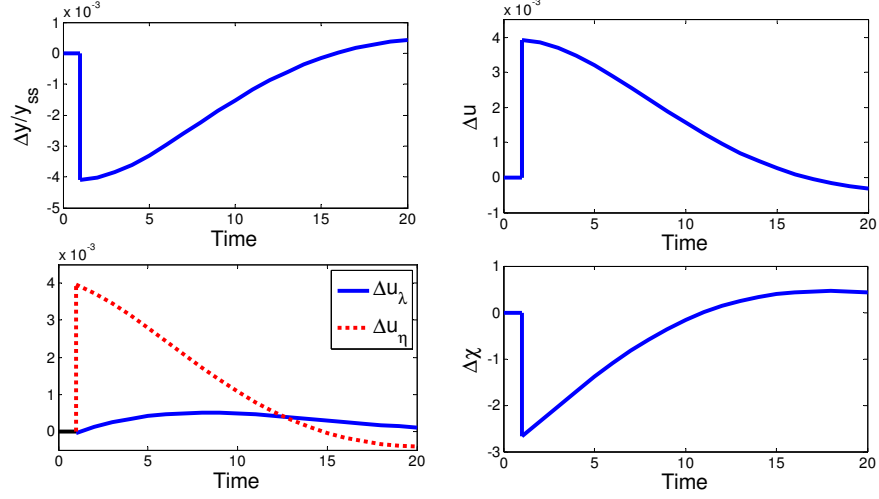
$\sigma_\lambda = 0.0032$	$\sigma_\eta = 0.0041$	$\sigma_z = 0.0051$
$\rho_{\lambda z} = -0.0644$	$\rho_{\lambda\eta} = 0.0899$	$\rho_{z\eta} = -0.4322$

Based on $\{\beta_{ij}\}$ and Σ , I revisit the impulse response exercise with correlations on the shocks. In each exercise, the level of the initial shock is set as σ_i with $i \in \{\lambda, \eta, z\}$. Then the shocks proceed with the VAR matrix $\{\beta_{ij}\}$. I document the key results in Figure (1.12). As illustrated in the upper and the middle panel, the shocks to the credit and labor markets generates co-movement of the recovery on output and employment. Therefore $\left(\frac{y_t - y_{ss}}{y_{ss}}, u_t - u_{ss}\right)$ is characterized with a perfectly negatively relationship in the associated figures. In contrast, the shock to aggregate productivity produces a gap (4 quarters) between the recovery of output and that of employment, which is demonstrated in the lower panel.

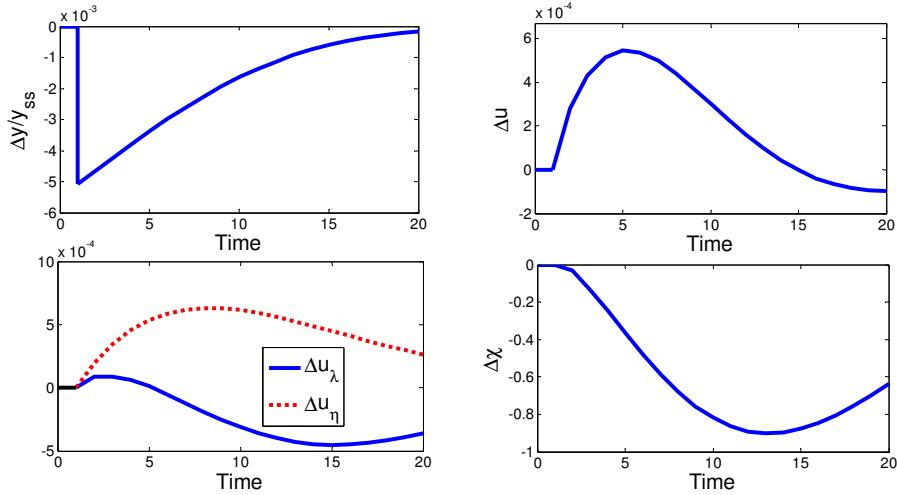
When it comes to recessions before the 1990s, the recovery on output and employment/unemployment occurred at the same pace. However, as shown in Figure (1.13), there exist gaps between output and employment/unemployment recovery in the past three recessions. It is typically labeled as jobless or sluggish recovery in the literature. The key message from Figure (1.12) is that the aggregate productivity shock is more likely to generate recovery gaps and thus more responsible for explaining the jobless or sluggish recovery in the recent three recessions. The shocks to the credit and labor markets, on the contrary, synchronize the recovery of output and employment/unemployment, and are in line with recessions prior to the 1990s. The intuition is, λ and η affect TFP and the equilibrium matching efficiency at the same time, which in turn determines output and unemployment. However, z only exerts an influence on TFP , but does not directly relate to the labor market. Consequently, the recovery of z immediately improves output while it alleviates unemployment only through increasing capital in the next few periods. In turn the z -shock produces a gap between output and employment recovery.



(a) IRF by λ -shock , with innovation magnitude σ_λ



(b) IRF by η -shock , with innovation magnitude σ_η



(c) IRF by z -shock , with innovation magnitude σ_z

Figure 1.12: Impulse Response of Shocks with Correlations

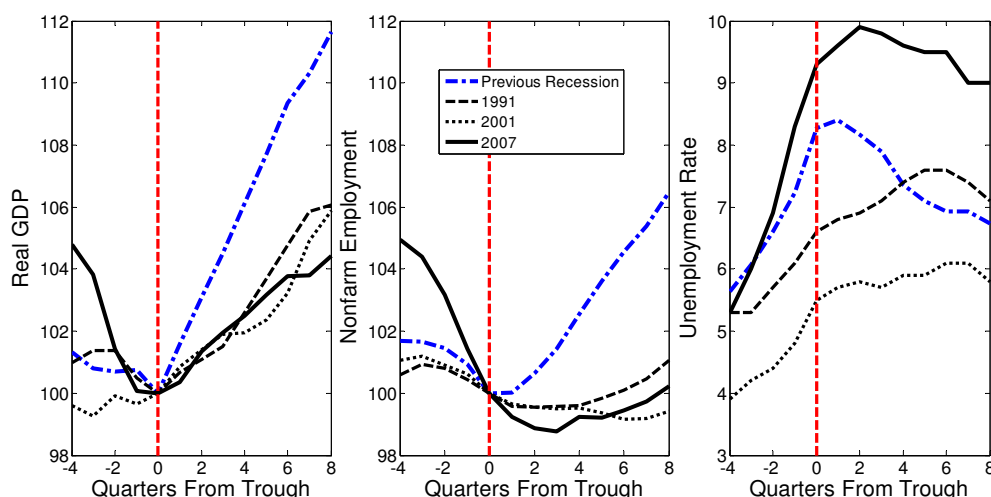


Figure 1.13: **Real GDP, Non-farm Private Employment and Unemployment Rate. Data Source: Bureau of Economic Analysis and Bureau of Labor Statistics**

In the end, taking into account the correlation between shocks still confirms the key insights obtained from the previous exercise on impulse response. That is, the shocks to the credit and labor markets increase and decrease the importance of credit imperfections in explaining a rising unemployment. The aggregate productivity shock exerts a lagged influence on unemployment compositions and helps emphasize the role of labor search frictions themselves in affecting unemployment. The patterns are more non-linear purely because of the correlations and persistence of these shocks.

1.4.3 Unemployment Decomposition over the Cycles

Now I initiate the unemployment decomposition. This subsection presents an analysis over all cycles between 1951Q4 and 2011Q4. The next part engages in a discussion for the recent financial crisis.

Regressions

In the spirit of Fujita and Ramey (2010) and Hobijn *et al.* (2012), I start our analysis with regressions. On the one hand, I have data on unemployment u_t . On the other

hand, I have recovered data on the shocks (λ_t, η_t, z_t) . I summarize the results in Table (1.2).

	u_t	$\log(u_t)$	$\log(u_t^{\text{predicted}})$
$\log(\lambda_t)$	-0.213**	-4.122**	-5.204**
$\log(\eta_t)$	-0.551**	-11.048**	-13.696**
$\log(z_t)$	-0.179**	-3.615**	-4.525**
<i>Adjusted - R²</i>	0.901	0.875	0.886
** : $p < 0.05$, ** : $p < 0.01$; constant is controlled.			

Table 1.2: **OLS Regression of Unemployment on Three Shocks**

The first and second columns use u and $\log(u)$ as the dependent variables, respectively. Thus the regression results denote the semi-elasticity and the elasticity of the shocks to unemployment, respectively. First, both regressions suggest a negative relationship between unemployment and the shocks. For example, the 1% increase of λ decreases unemployment by 0.0021, or by 4.1%. Second, the coefficient of η is largest since it directly relates to the matching efficiency in the labor market by definition. The effect of λ is larger than that of z since the former directly affects the matching efficiency while the latter does not. I return to the results in the third column after finishing the decomposition over the cycles in the next section.

Decomposition over the Cycles

The negative correlation between u and λ in Table (1.2) offers us a quick glimpse of contribution by credit imperfections to unemployment. To sharpen the analysis, I further explore the relationship between χ and the shocks. As already shown in the impulse response, I can recover unemployment compositions $\{u_t^\lambda, u_t^\eta\}$ and thus obtain $\chi_t \equiv u_t^\lambda / u_t$. In turn I reach Figure (1.14), a scatter plot immediately suggesting a negative and positive relationship between χ and (λ, η) respectively. Moreover, χ is negatively related to z , but not as significantly as with λ . This observation is consistent with the impulse response in the lower panels of Figures (1.11) and (1.12).

Moreover, like the regressions in Table (1.2), I obtain Table (1.3) by regressing χ on these shocks. The contribution of credit imperfections to unemployment rate does increase with η while it decreases with λ . This is true and is always statistically significant under all the robustness checks in the table. The findings are in line with

the theoretical predictions in Sections 3.2 and 3.3. However, the coefficient of z is not robust, as shown in the third and the fifth columns. What is reassuring is the effect of z on χ is relatively small, which is also consistent with the impulse response exercise in the previous parts.

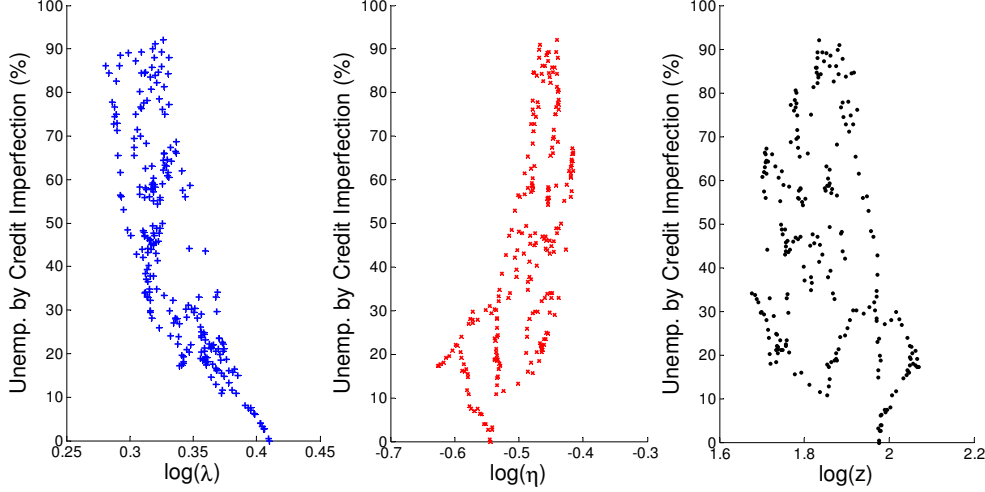


Figure 1.14: **Log-explanation of Credit Imperfections $\log(\chi)$ Versus Shocks**

In sum, the negative shock to the credit market increases the role of credit imperfections in explaining unemployment. It is just the opposite in the presence of a negative shock to the labor market or to aggregate productivity. Given the back-out series of (λ, η, z) , I demonstrate u^λ and χ , respectively, in the left and right panels of Figure (1.15). As implied in Figure (1.9), all the shocks are procyclical, in particular for λ and η . Consequently, it could be ambiguous whether χ is procyclical or not. I illustrate this argument in Figure (1.15). In the upper panel, I present the data, the predicted series and the predicted unemployment without credit imperfections. The lower panel in turn documents the series of χ over the cycles and suggests that χ is counter-cyclical with the real data. That is, even though credit crunches tend to increase χ , the simultaneous worsening labor market itself in recessions dampens the competing explanatory power of credit imperfections. Finally, averaging $\{\chi_t\}$ over the cycles suggests credit imperfections and the labor search frictions contribute around 46% and 54%, respectively, to unemployment.

$\log(\chi_t)$: log-explanation by credit imperfections for unemployment					
$\log(\lambda_t)$	-22.46**			-18.25**	-16.27**
$\log(\eta_t)$		9.77**		5.24**	10.36**
$\log(z_t)$			-2.92**		2.59**
<i>Adjusted - R²</i>	0.710	0.433	0.156	0.810	0.833
* $p < 0.05$; ** : $p < 0.01$; constant is controlled					

Table 1.3: **OLS Regression of $\log(\chi_t)$ on $\{\log(\lambda_t), \log(\eta_t), \log(z_t)\}$**

Decomposition in the Steady State

I have so far engaged in unemployment decomposition for each period over the cycles. I now move on to decompose unemployment in the steady state. As a review, I list the theoretical results as below.

$$\begin{aligned}
u_{ss} &= 1 - [\Omega(\lambda_{ss}) \cdot \eta_{ss}]^{\frac{1}{1-\gamma}} \cdot \left[\frac{\gamma \cdot ALP(\lambda_{ss}, z_{ss})}{1/\beta - 1 + \delta} \right]^{\frac{\gamma}{1-\gamma}} \\
u_{ss}^* &= 1 - [\Omega(\lambda^*) \cdot \eta_{ss}]^{\frac{1}{1-\gamma}} \cdot \left[\frac{\gamma \cdot ALP(\lambda^*, z_{ss})}{1/\beta - 1 + \delta} \right]^{\frac{\gamma}{1-\gamma}}
\end{aligned}$$

Replacing λ_{ss} with $\bar{\lambda} = \max\{\lambda_t\}$ suggests that, the contribution of credit imperfections to unemployment, $\frac{u_{ss} - u_{ss}^*}{u_{ss}}$, is 52%. I can further decompose the explanatory power by credit imperfections into the extensive and intensive margins, which are 98% and 2% respectively. Notice the explanatory power in the steady state, 52%, is larger than the average explanatory power over the cycles mentioned above (46%). This is due to the fact that I treat capital as given when calculating χ_t over the cycles while capital accumulation is endogenous and is positively related with λ in the steady state. If I use ∞ rather than $\max\{\lambda_t\}$ as the limit case, then the importance of credit imperfections in unemployment would be higher.

1.4.4 Unemployment Decomposition in the Recent Financial Crisis

As shown in the left panel of Figure (1.17), the last quarter of 2008 experienced a significant credit crunch. I use our model to address the consequence of the credit crunch for the labor market. More specifically, how much does the credit crunch add to the outward shift in the Beveridge curve in the recent financial crisis? I illustrate the controlled experiment in Figure (1.16). The solid lines in these three panels denote the recovered shocks since the last quarter of 2008. The dashed line

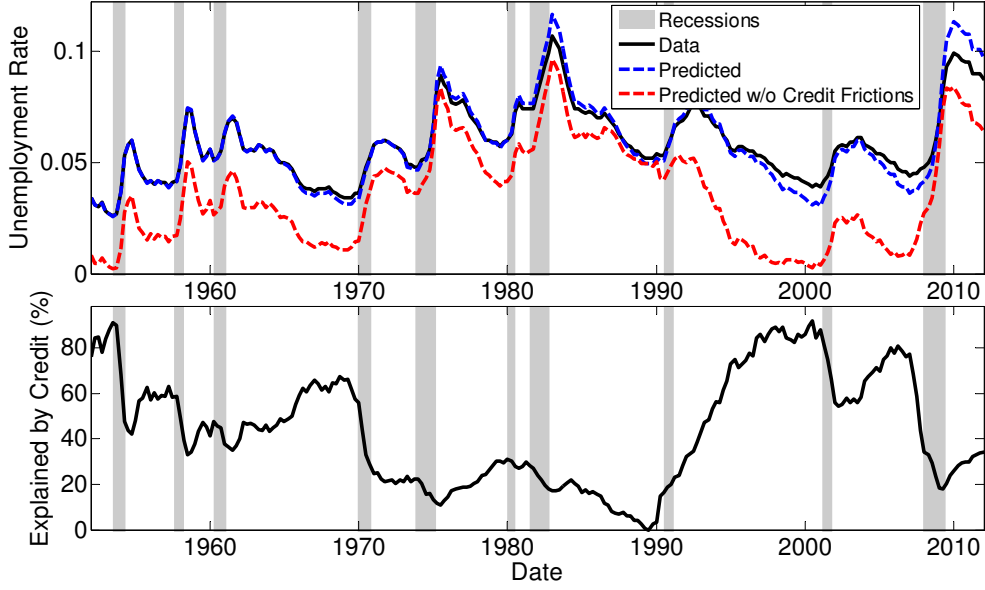


Figure 1.15: **Unemployment: Data, the Model-Predicted, and that without Credit Frictions**

in the left panel denotes the counter-factual shock to the credit market, which is held constant at the level just before the credit crunch. Equivalently, I show the controlled shock to the credit market in the left panel of Figure (1.17).

Then I simulate the economy with capital accumulation being endogenous, which is governed by the transition dynamics in Section 3. I summarize the counter-factual dynamics of unemployment in the middle panel of Figure (1.17). The difference between the data and the counter-factual is interpreted as unemployment contributed by the credit crunch. Taking the average yields that the credit crunch contributes around 26.7% of unemployment in this financial crisis. Notice that the number is lower than that over the cycles. I have shown $\frac{\partial x}{\partial \eta} < 0$ in Section 3.3. Meanwhile, the middle panel of Figure (1.16) reveals that in the past recession η_t also decreases, *i.e.*, the labor market itself has also received a negative shock. It in turn attenuates the power of negative credit shocks in explaining unemployment in recessions.

Alternatively, I can use the results already established in unemployment decomposition over the cycles. In particular, I focus on the decomposition since the last quarter of 2008 in Figure (1.15), which is documented in the right panel of Figure (1.17). A calculation suggests that credit imperfections accounts for around 27.4% for the recent recession. Notice that the quantitative results are very similar to each other. Therefore, I claim that the credit crunch contributes 27% $(=(27.4\%+26.7\%)/2)$ for the recent financial crisis.

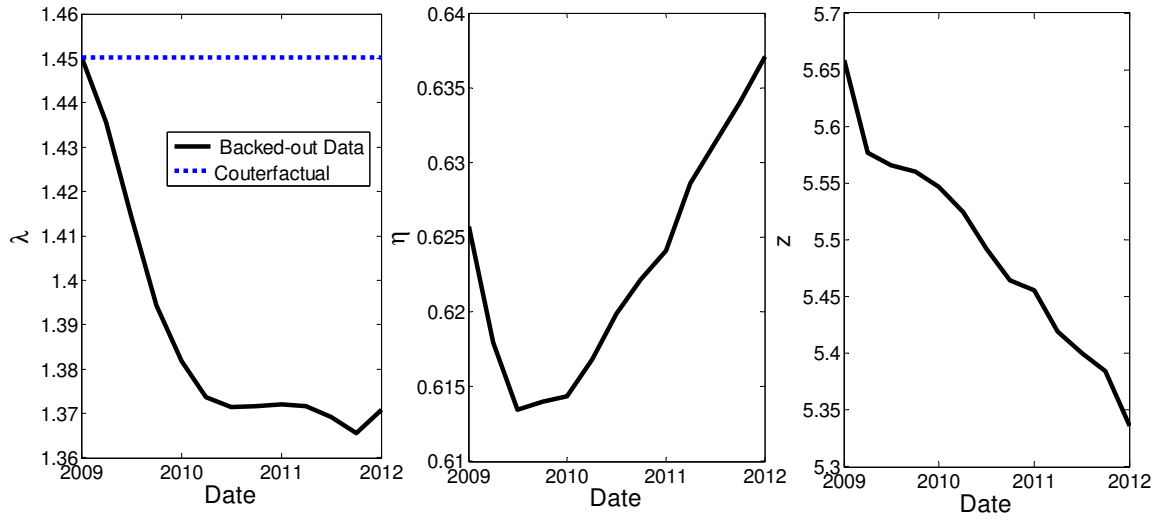


Figure 1.16: **Corresponding Shocks for the Counter-factual Analysis for the 2008 Financial Crisis**

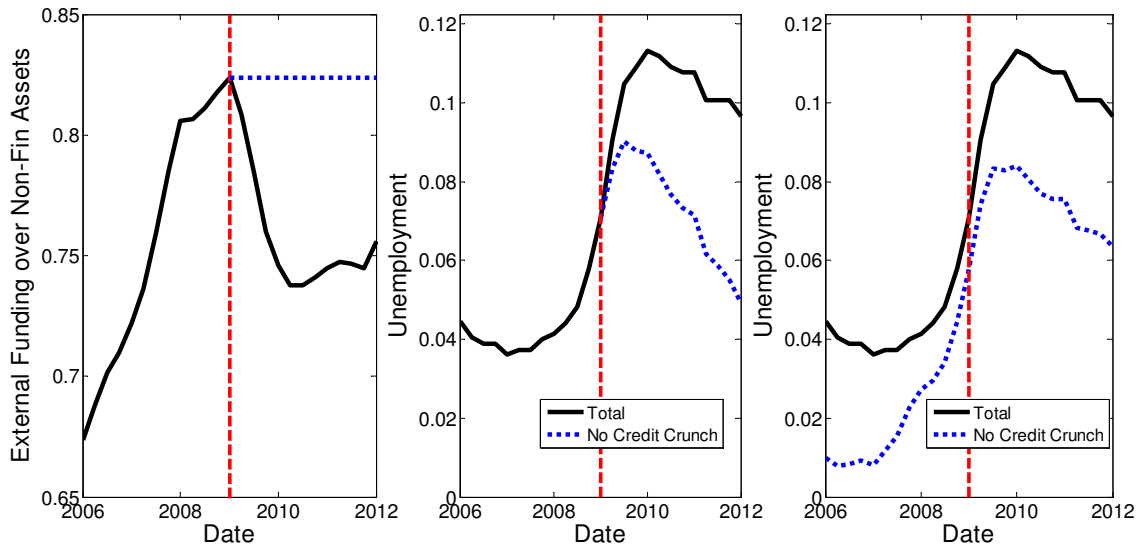


Figure 1.17: **Counter-factual Analysis for the Recent Financial Crisis**

Finally, I reach Figure (1.18) by combining the model-predicted unemployment with vacancy data in JOLTS. The left panel suggests that data and the model fit well with each other. The right panel describes the path of the Beveridge curve if there was no credit crunch in the past financial recession. Notice that unemployment in the right panel continues to rise although I unplug the negative shock to the credit market. Unemployment in the counter-factual analysis is then purely driven by the negative shocks to the labor market and to aggregate productivity, as indicated by the middle and right panels of Figure (1.17). The most intriguing finding is that the counter-factual Beveridge curve does not shift outward, but instead moves alongside the original curve prior to the financial crisis. Therefore, the credit crunch seems to be mainly responsible for the outward shift in the Beveridge curve in the recent financial crisis.

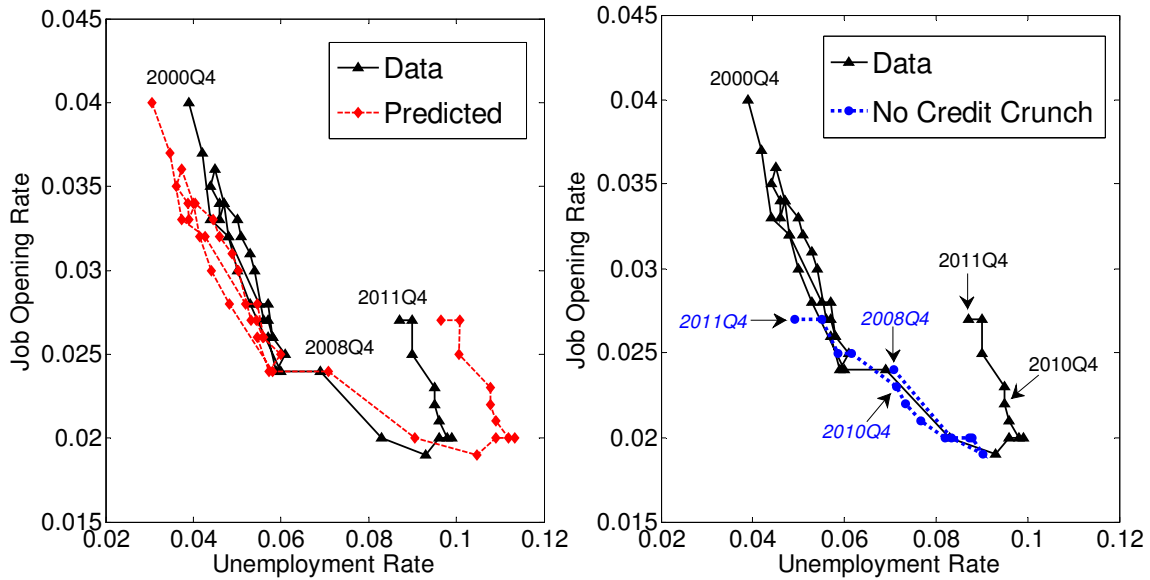


Figure 1.18: **Left Panel: Data and Model-Predicted Beveridge Curve; Right Panel: Data and Model-Predicted Beveridge without Credit Crunch in 2008**

1.5 Which Shocks Are Most Essential

I have so far exclusively addressed the aggregate implications of the model, especially for the unemployment rate. However, the transmission mechanism works through the heterogeneous agents at the micro level. Therefore, this section is devoted to discussing the heterogeneous treatment effect of the aggregate shocks on firms. In

particular, I confront the disaggregate implications of these three shocks with micro-level empirical findings.

Employment Distribution

Moscarini and Postel-Vinay (2012) suggest that large firms have a more significant response to employment than do small firms in recessions. In our paper, the share of employment by firms with individual productivity no smaller than $\varphi^\omega = x^\omega/z$ is given as follows,

where φ^ω is denoted as the upper- ω percentile, *i.e.*, $1 - F(\varphi^\omega) = \omega$. The employment share by firms with productivity no smaller than φ^ω is illustrated in Figure (1.19). I can check that $\Gamma(\varphi^\omega)$ has nothing to do with matching efficiency η or aggregate productivity z , but only increases with financial friction λ . I illustrate the effect of λ on $\Gamma(\varphi^\omega)$ in Figure (1.19). Alternatively, the job-filling rate $q(\varphi)$ increases with φ . Therefore, the employment loss increases with φ when a credit crunch occurs.

$$\Gamma(\varphi^\omega) = \frac{\int_{\max\{\varphi^\omega, \hat{\varphi}\}}^{\bar{\varphi}} q(\varphi) \cdot dF(\varphi)}{\int_{\hat{\varphi}}^{\bar{\varphi}} q(\varphi) \cdot dF(\varphi)} = 1 - F^M(\varphi^\omega),$$

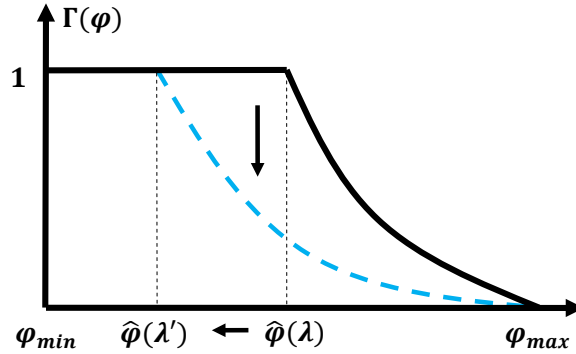


Figure 1.19: **Employment Shares**

Productivity Dispersion

The empirical research by Kehrig (2011) suggests the productivity dispersion widens in recessions. Firm productivity in our paper is the product of aggregate productivity and the individual component, *i.e.*, $x = z \cdot \varphi$. I can verify that the shock to the labor market (η) is related to neither the productivity distribution of the incumbents nor that of the new-entry firms. Instead, the negative shock to the credit market (λ) and

that to aggregate productivity (z) widens and shrinks the productivity distribution of the incumbents and that of the new-entry firms, respectively.

	$z \downarrow$	$\eta \downarrow$	$\lambda \downarrow$
employment share of productive firms	\rightarrow	\rightarrow	\downarrow
productivity dispersion	\downarrow	\rightarrow	\uparrow

Table 1.4: **Aggregate Implications of These Shocks**

I summarize the disaggregate implications of the aggregate shocks in Table (1.4). It seems the prediction by credit shock coincides with the empirical regularity set by Kehrig (2011) and Moscarini and Postel-Vinay (2012). Moreover, the aggregate shocks have different aggregate implications, as shown in Table (1.5).³²

	D/K	ALP	TFP	Y	u	χ	RP
$\lambda \downarrow$	\downarrow	\downarrow	\downarrow	\downarrow	\uparrow	\uparrow	\uparrow
$z \downarrow$	\rightarrow	\downarrow	\downarrow	\downarrow	\rightarrow	\rightarrow	\downarrow
$\eta \downarrow$	\rightarrow	\rightarrow	\downarrow	\downarrow	\uparrow	\downarrow	\uparrow

Table 1.5: **Aggregate Implications of These Shocks**

1.6 Conclusion

I develop a tractable dynamic model with heterogeneous entrepreneurs to explain the interaction between credit and labor markets. The model is used to characterize the implications of capital misallocation due to credit imperfections for unemployment. The marginal value of being matched with labor increases with an entrepreneur's level of productivity. Therefore, high-productivity entrepreneurs offer a higher wage in equilibrium and thus the job-filling rate increases with their productivity. A credit crunch worsens capital misallocation by redistributing capital from high-productivity to low-productivity entrepreneurs. The former group of entrepreneurs is not only more productive given that capital is matched with labor, but also they are better at being matched with labor. Consequently, a credit crunch lowers aggregate matching efficiency in the labor market. This is the key theoretical contribution of this paper.

I then quantify the unemployment effect of credit imperfections after a calibration to the US economy. First, the exercise on unemployment decomposition reveals that

³²As defined in Section 2, the return premium is defined as $RP \equiv \mathbb{E} [\max (\pi (\varphi) - r, 0)]$. I can rewrite it as $RP = \left(\frac{\tau(\lambda)}{\lambda} \right) \cdot \left(\frac{\partial Y}{\partial K} \right)$ and thus decreases with λ while increases with η and z .

credit imperfections and the classic labor search frictions contribute around 46% and 54%, respectively, to unemployment over the cycles between 1951 and 2011. Second, I conduct a counter-factual analysis to show that the credit crunch serves as the key driving force behind the outward shift in the Beveridge curve in the recent recession.

Another quantitative analysis is to address the jobless/sluggish recovery. On the one hand, the impulse response after VAR implies that the shock to aggregate productivity is more likely to be a cause for the jobless or sluggish recovery in the recent three recessions. On the other hand, the shocks to the credit and labor markets seem more responsible for the simultaneous recovery pattern between output and unemployment in recessions prior to the 1990s. Finally, I confront the disaggregate prediction of our model with micro-level empirical findings. Credit shocks are seemingly most essential to explain the widening productivity distribution and the dis-proportional loss of employment in recessions.

I close with several promising directions for further research. First, as documented by Hobijn and Sahin (2012), the outward shift in the Beveridge curve since the Great Recession occurred not only in the US, but also in other OECD countries like Portugal, Spain, and the UK. Like Hsieh and Klenow (2007), and Restuccia, Yang and Zhu (2008), it could be interesting to initiate a cross-country analysis for the quantitative effect of credit crunches on unemployment. Second, I have exclusively focused on the effect of credit imperfections on unemployment. As shown in Section 3.4, I take a preliminary step related to the implications of an MPS of the productivity distribution for unemployment. It may be fruitful to combine our work with that of Bigio (2013). Third, I have used a static wage contract throughout the paper for tractability. It could be challenging but fruitful to address the same problem with a dynamic labor contract as well as with credit imperfections and endogenous capital accumulation. The recent progress on labor search with heterogeneity by Menzio and Shi (2010, 2011), Schaal (2012) and Kaas and Kircher (2013) may serve as a reasonable starting point. Finally, the flexible framework of our paper could offer a tractable tool for us to model and quantify the role of unconventional monetary policy in curing the high rate unemployment. See the recent work by Dong and Wen (2013) for an example.

1.7 Appendix

1.7.1 Appendix A - Data Sources, Definitions and Calculations

All the data throughout this paper are of quarterly frequency. There are three sources of data used in Section 1 and Section 4. First, I use financial data from the Flow of Funds Accounts (FFA) to construct the ratio of external funding over non-financial assets. I follow exactly Buera, Fattal-Jaef and Shin (2013) for this measurement. On the one hand, the external funding corresponds to the credit market instruments in FFA. It consists of the bank loans of the corporate and non-corporate sectors, and the commercial papers, corporate bonds and municipal securities of the corporate business. On the other hand, non-financial assets include real estate stock, equipment, software and inventories of the corporate and non-financial non-corporate business.

Second, data on employment, unemployment rate and job creation/destruction come from Bureau of Labor Statistics (BLS) while data on the Beveridge curve Job Opening and Labor Turnover (JOLTS). I only consider employment by non-farm private sectors. I use unemployment rate and employment to recover the total labor participation numbers in non-farm private sectors. The Beveridge curve with job opening rate and unemployment rate started with the last of 2000 because that is the starting point of the data in JOLTS.

Finally, National Income and Product Account (NIPA) documents quarterly data on output and investment, and annual data on capital. Output is defined as the sum of private non-durable consumption and private non-residential investment. I use the quarterly data on investment and the annual data on capital to recover the quarterly data on capital.

1.7.2 Appendix B - A Static Simplified Model

I use a static and simplified model to illustrate the key mechanism through which credit misallocation lowers aggregate matching efficiency. Aggregate productivity is

simply set as $z = 1$. Each entrepreneur has K units of net worth. The distribution of individual productivity is a simple Binomial, *i.e.*, φ adopts $\varphi_H = \mu + \sigma$ and $\varphi_L = \mu - \sigma$ with equal probability, where $\sigma \in [0, \mu]$. As in the baseline, I model credit and labor frictions by a collateral constraint and competitive search respectively. I first characterize the case with only labor search frictions.

$$Y^* = \max \{ \varphi_H \cdot m(v_H, l_H) + \varphi_L \cdot m(v_L, l_L) \}$$

subject to

$$\begin{aligned} v_H + v_L &\leq K \\ l_H + l_L &\leq L \\ v_i, l_i &\geq 0, \quad i \in \{L, H\}, \end{aligned}$$

where v_i and l_i denotes respectively the measure of capital and labor in sub-labor market $i \in \{L, H\}$, and $m(\cdot, \cdot)$ a matching technology. The efficient allocation consists of $v_H^* = K$, $v_L^* = 0$, $l_H^* = L$, and $l_L^* = 0$. In turn, aggregate output is $Y^* = \varphi_H \cdot m(K, L)$, employment $N^* = m(K, L)$, average labor productivity $ALP^* \equiv \frac{Y^*}{N^*} = \varphi_H$, and unemployment $u^* \equiv 1 - \frac{N^*}{L}$. Then I reach the equilibrium allocation as below.

Corollary 6. (Equilibrium Wedges under a Simple Binomial Distribution)

Denote \tilde{F} as a Binomial distribution such that φ adopts φ_H and φ_L with probability $\alpha_H(\lambda)$ and $1 - \alpha_H(\lambda)$ respectively, where $\alpha_H(\lambda) \equiv \min \left\{ \frac{\lambda}{2}, 1 \right\}$. Then

1. for $i \in \{L, H\}$, the total capital used by type- i entrepreneurs is

$$v_H = \min \left\{ \frac{\lambda}{2}, 1 \right\} \cdot K, \quad v_L = K - v_H.$$

2. aggregate output and employment is

$$Y = (1 - \tau_y) \cdot Y^*, \quad N = (1 - \tau_n) \cdot N^*,$$

where

$$\begin{aligned}
1 - \tau_y &= \Lambda(\lambda) \equiv \frac{\left[\mathbb{E}_{\tilde{F}}\left(\varphi^{\frac{1}{\gamma}}\right)\right]^{\gamma}}{\varphi_H} = \left[\alpha_H(\lambda) + (1 - \alpha_H(\lambda)) \cdot \left(\frac{\varphi_L}{\varphi_H}\right)^{\frac{1}{\gamma}}\right]^{\gamma} \\
1 - \tau_n &= \Omega(\lambda) \equiv \frac{\mathbb{E}_{\tilde{F}}\left(\varphi^{\frac{1-\gamma}{\gamma}}\right)}{\left[\mathbb{E}_{\tilde{F}}\left(\varphi^{\frac{1}{\gamma}}\right)\right]^{1-\gamma}} = \frac{\alpha_H(\lambda) + (1 - \alpha_H(\lambda)) \cdot \left(\frac{\varphi_L}{\varphi_H}\right)^{\frac{1-\gamma}{\gamma}}}{\left[\alpha_H(\lambda) + (1 - \alpha_H(\lambda)) \cdot \left(\frac{\varphi_L}{\varphi_H}\right)^{\frac{1}{\gamma}}\right]^{1-\gamma}}.
\end{aligned}$$

both of which increases with λ , decreases with $\frac{\sigma}{\mu}$, $\lim_{\lambda \rightarrow \infty} \tau_y = \lim_{\lambda \rightarrow \infty} \tau_n = 0$, and

$$\lim_{\frac{\sigma}{\mu} \rightarrow 0} \tau_y = \lim_{\frac{\sigma}{\mu} \rightarrow 0} \tau_n = 0.$$

Similar to Proposition 2, a credit crunch increases the wedge of output and employment. Moreover, an MPS of the productivity distribution, *i.e.*, the increase of $\frac{\sigma}{\mu}$, also lowers aggregate matching efficiency. I use Figure 1.20 to illustrate those findings.

The main merit of using a Binomial distribution is a more clear intuition behind the transmission mechanism from credit to labor markets. By definition, employment is $N \equiv m(v_H, l_H) + m(v_L, l_L) = v_H \cdot q_H + v_L \cdot q_L$, where q_i denotes the job-filling rate in sub-labor market i . To make the analysis non-trivial, I assume both sub-labor markets are active, *i.e.*, $v_H > 0$ and $v_L > 0$. Then I have

$$\frac{\partial N}{\partial \lambda} = (q_H - q_L) \cdot \frac{\partial v_H}{\partial \lambda} + \left(v_H \cdot \frac{\partial q_H}{\partial \lambda} + v_L \cdot \frac{\partial q_L}{\partial \lambda}\right) \geq 0.$$

I have shown that $w_H > w_L$, $\theta_H > \theta_L$, and $q_H > q_L$. Then as the above decomposition suggests, on the one hand, the increase of λ transfers capital from low-productivity to high-productivity entrepreneurs, which directly implies an increase of employment. On the other hand, the increase of λ makes the use of capital more congested and thus the job-filling rate in the active sub-labor markets decrease. However, the direct effect can be verified to dominate the indirect general-equilibrium effect.

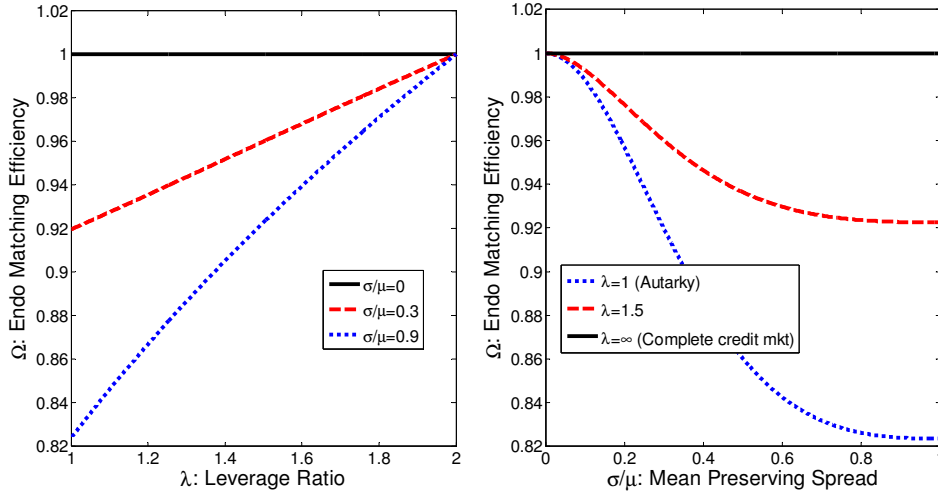


Figure 1.20: **Left Panel:** $(\Omega, \frac{\sigma}{\mu})$; **Right Panel:** (Ω, λ)

1.7.3 Appendix C - Model Extension

This section consists of three pieces of model extension.³³ The first two extensions consider other possible sources of capital misallocation. One is to introduce tax on capital revenue while the other is to address the implications of working capital constraint. For space concern, I omit the discussion on transition dynamics. Finally, motivated by recent empirical findings, I endogenize firm's procyclical recruiting effort, which in turn amplifies the transmission mechanism in our baseline.

Tax on Capital Revenue

Motivated by Restuccia and Rogerson (2008), I extend the model with a tax scheme on capital revenue $\{\tau_k(\varphi)\}_{\varphi \in \Phi}$.³⁴ The expected capital revenue is then adjusted as $\tilde{\pi}(\varphi) = [1 - \tau_k(\varphi)] \cdot \pi(\varphi)$. Meanwhile, the active set is updated as $\Phi_A = \{\varphi | \tilde{\pi}(\varphi) \geq r\}$ with its associated cumulative distribution as F_A and the lower bound as $\hat{\varphi} = \inf\{\Phi_A\}$. In the baseline, I characterize capital misallocation by the decrease of the cut-off point $\hat{\varphi}$. I generalize the notion of capital misallocation as follows.

Definition 2. Denote F_A and $F_{A'}$ as two pieces of productivity distribution of active entrepreneurs. $F_{A'}$ causes a worse capital misallocation than F_A if and only if F_A

³³For space concern, I remove all the proofs associated with this section. The proofs are available upon request.

³⁴For simplicity, I assume entrepreneurs with the same productivity share the same tax rate.

Second-Order Stochastic Dominates (SOSD) $F_{A'}$.

The employment in Equation (1.19) is now generalized by

$$N(F_A) = \mathbb{E}_{F_A} [q(\varphi, W)] \cdot K = \left[\int_{\Phi} q(\varphi, W(F_A)) dF_A \right] \cdot K \quad (1.22)$$

where K denotes the aggregate capital supply, $q(\varphi)$ the job-filling rate in sub-labor market φ and W the expected labor revenue. As in the baseline, capital misallocation generates two competing effects on employment. I illustrate the generalized version as below.

$$N(F_A) - N(F_{A'}) = \left\{ \left[\int_{\Phi} q(\varphi, W(F_A)) \cdot dF_A - \int_{\Phi} q(\varphi, W(F_A)) \cdot dF_{A'} \right] + \left[\int_{\Phi} q(\varphi, W(F_A)) \cdot dF_{A'} - \int_{\Phi} q(\varphi, W(F_{A'})) \cdot dF_{A'} \right] \right\} \cdot K.$$

To sharpen the analysis, I make an assumption as below, which delivers the generalized version of the unemployment effect of capital misallocation.

Assumption 2. The distribution F is specified such that, if the truncated distribution F_A SOSD $F_{A'}$,

$$\frac{\mathbb{E}_{F_A} \left(\varphi^{\frac{1-\gamma}{\gamma}} \right)}{\left[\mathbb{E}_{F_A} \left(\varphi^{\frac{1}{\gamma}} \right) \right]^{1-\gamma}} > \frac{\mathbb{E}_{F_{A'}} \left(\varphi^{\frac{1-\gamma}{\gamma}} \right)}{\left[\mathbb{E}_{F_{A'}} \left(\varphi^{\frac{1}{\gamma}} \right) \right]^{1-\gamma}}.$$

Corollary 7. (Wedges with Capital Revenue Tax) Under Assumption 2, if F_A SOSD $F_{A'}$,

1. the wedges to aggregate output and employment are

$$\begin{aligned} 1 - \tau_y &\equiv \left\{ \mathbb{E}_{F_A} \left[\left(\varphi / \bar{\varphi} \right)^{\frac{1}{\gamma}} \right] \right\}^{\gamma} \in [0, 1] \\ 1 - \tau_n &\equiv \frac{\mathbb{E}_{F_A} \left(\varphi^{\frac{1-\gamma}{\gamma}} \right)}{\left[\mathbb{E}_{F_A} \left(\varphi^{\frac{1}{\gamma}} \right) \right]^{1-\gamma}} \in [0, 1] \end{aligned}$$

where (τ_y, τ_n) are larger with $\Phi_{A'}$.

2. the wedges to ALP and unemployment are

$$\begin{aligned} 1 - \tau_{alp} &= \frac{1 - \tau_y}{1 - \tau_n} = \frac{\mathbb{E}_{F_A} \left(\varphi^{\frac{1}{\gamma}} \right)}{\mathbb{E}_{F_A} \left[\left(\frac{\bar{\varphi}}{\varphi} \right) \cdot \varphi^{\frac{1}{\gamma}} \right]} \in [0, 1] \\ 1 + \tau_u &= 1 + \tau_n \cdot \left(\frac{1 - u^*}{u^*} \right) \in (1, \infty), \end{aligned}$$

3. the wedge to the expected wage revenue is zero while that to the interest rate is

$$1 - \tau_r = \frac{1 - \tau_k(\hat{\varphi})}{\mathbb{E}_{F_A} \left[\left(\frac{\varphi}{\hat{\varphi}} \right)^{\frac{1}{\gamma}} \right]} \in [0, 1]$$

On the one hand, if $\tau_K(\varphi) \equiv 0$, then the active sets reduces to that in the baseline. On the other hand, if $\tau_K(\varphi)$ is progressive, taking $\tau_K(\varphi) = \alpha \cdot \left[1 - \left(\frac{\varphi - \underline{\varphi}}{\bar{\varphi} - \underline{\varphi}} \right)^{\frac{1}{\gamma}} \right]$ with $\alpha \in [0, 1]$ for example, then the active set is $\Phi_A = \{\varphi | \varphi \in [\hat{\varphi}_1, \hat{\varphi}_2]\}$, which is illustrated in Figure 1.21. Moreover, I show that the increase of α widens the active set Φ_A and thus lowers the output and increases unemployment.

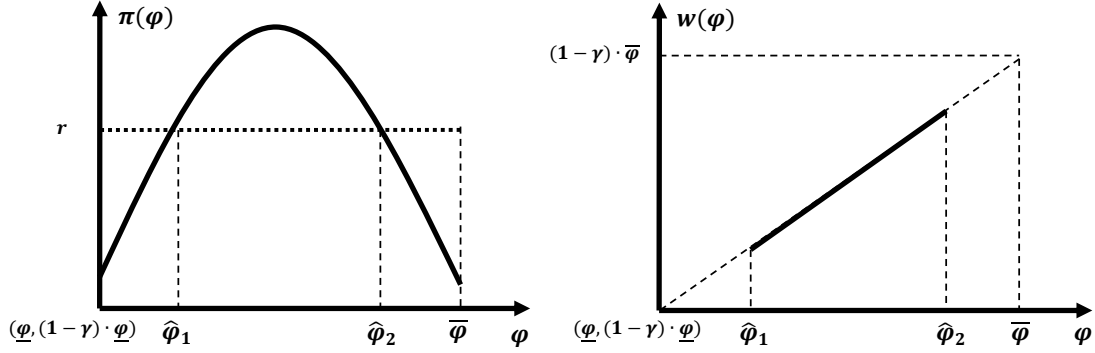


Figure 1.21: **Wage Scheme with a Progressive Tax on Capital Revenue (an example)**

Working Capital Constraint

Hosios condition is satisfied in the baseline with competitive search. Therefore the labor wedge is zero in the baseline. However, the business cycle accounting by Chari, Kehoe and McGrattan (2007) suggests the quantitative importance of labor wedge. This part imposes a working capital constraint upon the baseline to produce a non-trivial labor wedge. As shown in Section 2, total wage payment is $k \cdot \theta(\varphi) \cdot W$ for entrepreneurs with productivity φ and with k units of capital for production. I assume entrepreneurs have to pay part of the wage bill before production such that $k \cdot \theta(\varphi) \cdot W \leq \lambda_w \cdot k$, or equivalently,

$$\theta(\varphi) \leq \frac{\lambda_w}{W}. \quad (1.23)$$

In contrast to the baseline, the equilibrium wage scheme may be distorted in the presence of a constraint on working capital. The following proposition characterizes the equilibrium wedges on productivity, employment, interest rate, and wages, etc in the presence of the working capital constraint.

Corollary 8. (Equilibrium Wedges with Working Capital Constraint) In each period,

1. there exist pairwise cut-off values $(\hat{\varphi}, \tilde{\varphi})$ such that
 - (a) only entrepreneurs with productivity $\varphi \geq \hat{\varphi}$ are active in production,
 - (b) the wage scheme is $w(\varphi) = (1 - \gamma) \cdot \min\{\varphi, \tilde{\varphi}\}$,
2. the solution to the pairwise cut-off values $(\hat{\varphi}, \tilde{\varphi})$ exists and is unique, and
 - (a) $\hat{\varphi}$ increases with λ and has nothing to do with other variables,
 - (b) $\tilde{\varphi}$ increases with λ , and λ_w ,
3. the wedges to aggregate output and employment are

$$\begin{aligned}
1 - \tau_y &\equiv \Lambda(\lambda, \lambda_w) \equiv \frac{\mathbb{E} \left\{ \max \left(1, \left(\frac{\varphi}{\bar{\varphi}} \right) \right) \cdot [\min(\varphi, \tilde{\varphi})]^{\frac{1}{\gamma}} \mid \varphi \in [\hat{\varphi}, \bar{\varphi}] \right\}}{\left[\mathbb{E} \left\{ [\min(\varphi, \tilde{\varphi})]^{\frac{1}{\gamma}} \mid \varphi \in [\hat{\varphi}, \bar{\varphi}] \right\} \right]^{1-\gamma} \cdot \bar{\varphi}} \in [0, 1] \\
1 - \tau_n &\equiv \Omega(\lambda, \lambda_w) \equiv \frac{\mathbb{E} \left\{ [\min(\varphi, \tilde{\varphi})]^{\frac{1-\gamma}{\gamma}} \mid \varphi \in [\hat{\varphi}, \bar{\varphi}] \right\}}{\left[\mathbb{E} \left\{ [\min(\varphi, \tilde{\varphi})]^{\frac{1}{\gamma}} \mid \varphi \in [\hat{\varphi}, \bar{\varphi}] \right\} \right]^{1-\gamma}} \in [0, 1]
\end{aligned}$$

and (τ_y, τ_n) decreases with λ_w .

4. the wedges to ALP and unemployment are

$$\begin{aligned}
1 - \tau_{alp} &= \frac{1 - \tau_y}{1 - \tau_n} = \frac{\mathbb{E} \left\{ \max \left(1, \left(\frac{\varphi}{\bar{\varphi}} \right) \right) \cdot [\min(\varphi, \tilde{\varphi})]^{\frac{1}{\gamma}} \mid \varphi \in [\hat{\varphi}, \bar{\varphi}] \right\}}{\mathbb{E} \left\{ [\min(\varphi, \tilde{\varphi})]^{\frac{1-\gamma}{\gamma}} \mid \varphi \in [\hat{\varphi}, \bar{\varphi}] \right\} \cdot \bar{\varphi}} \in [0, 1] \\
1 + \tau_u &= 1 + \tau_n \cdot \left(\frac{1 - u^*}{u^*} \right) \in (1, \infty).
\end{aligned}$$

and (τ_{alp}, τ_u) decreases with λ_w .

5. the wedges to the interest rate and to the wage are

$$1 - \tau_r = \frac{1}{\mathbb{E} \left\{ \min \left(\left(\frac{\varphi}{\underline{\varphi}} \right)^{\frac{1}{\gamma}}, \left(\frac{\tilde{\varphi}}{\underline{\varphi}} \right)^{\frac{1}{\gamma}} \cdot \left(\frac{\varphi}{\tilde{\varphi}} \right) \right) \mid \varphi \in [\hat{\varphi}, \bar{\varphi}] \right\}} \in [0, 1].$$

$$1 - \tau_w = \frac{\mathbb{E} \left\{ [\min(\varphi, \tilde{\varphi})]^{\frac{1}{\gamma}} \mid \varphi \in [\hat{\varphi}, \bar{\varphi}] \right\}}{\mathbb{E} \left\{ \max \left(1, \left(\frac{\varphi}{\underline{\varphi}} \right) \right) \cdot [\min(\varphi, \tilde{\varphi})]^{\frac{1}{\gamma}} \mid \varphi \in [\hat{\varphi}, \bar{\varphi}] \right\}} \in [0, 1].$$

and (τ_r, τ_w) decreases with λ_w .

If λ_w is high enough, then $\tilde{\varphi} > \bar{\varphi}$ and I am back to the baseline model. Otherwise, as indicated in the above corollary, the optimal wage scheme becomes flattened for $\varphi \in [\hat{\varphi}, \tilde{\varphi}]$. The wage scheme with a binding working capital constraint provides a micro foundation for equilibrium wage rigidity, *i.e.*, entrepreneurs choose not to adjust their wage scheme if their productivity $\varphi \in [\hat{\varphi}, \tilde{\varphi}]$. I illustrate it in Figure 1.22.

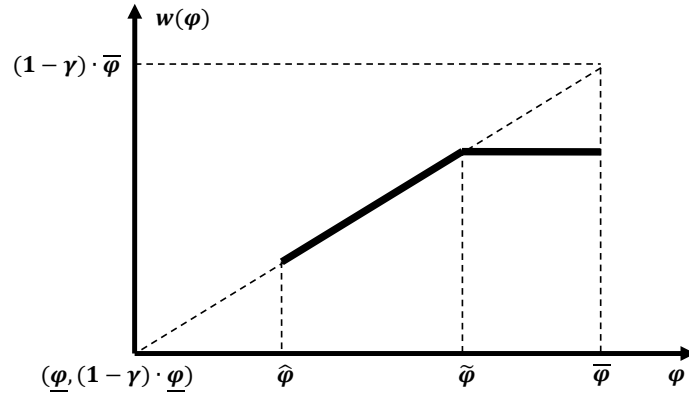


Figure 1.22: **Wage Scheme with Working Capital Constraint**

The possibility of non-trivial labor wedge is the main insight gained from the working capital constraint. The marginal value of being matched with labor increases with entrepreneur's productivity. Thus wage scheme and job-filling rate increase with productivity. However, due to the working capital constraint, high-productivity entrepreneurs would have to cut down the otherwise high wage, which in turn lowers employment and labor expenditure.

Endogenous Recruiting Effort

The recent empirical findings by Davis, Faberman and Haltiwanger (2012) suggest that, in addition to posting vacancies, firm's recruiting effort also includes "increase advertising or search intensity per vacancy, screen applicants more quickly, relax hiring standards, improve working conditions, and offer more attractive compensation to prospective employees".³⁵ Furthermore, they show that firm's recruiting effort is procyclical. To this end, I follow Pissarides (2000) and Bai, Ríos-Rull and Storesletten (2012) to endogenize firm's search effort.

For simplicity, I assume firm's search effort is made after observing the aggregate state variable, but before the realization of their own productivity level. Entrepreneurs use worker's labor input to increase their own search effort s , which may include advertising and screening effort.³⁶ More specifically, $\sigma \in (0, 1)$ of capital revenue is pledgeable to workers. Matching function is $m(v(\varphi)e(\bar{s}), l(\varphi))$, where \bar{s} denotes the average recruiting effort. Each entrepreneur treats \bar{s} as given. In equilibrium, I have $s = \bar{s}$. Denote the modified market tightness as $\theta(\varphi) \equiv \frac{l(\varphi)}{v(\varphi)e(\bar{s})}$. The job-filling rate and job finding rate are modified as below.

$$\begin{aligned} q(\theta(\varphi), s) &= \frac{m(v(\varphi)e(\bar{s}), l(\varphi))}{v(\varphi)e(\bar{s})} \cdot e(s) = m(1, \theta(\varphi)) \cdot e(s) \\ p(\theta(\varphi), s) &= \frac{m(v(\varphi)e(\bar{s}), l(\varphi))}{l(\varphi)} = m\left(\frac{1}{\theta(\varphi)}, 1\right) = \frac{q(\theta(\varphi), s)}{\theta(\varphi) \cdot e(s)}. \end{aligned}$$

Corollary 9. In equilibrium $s = \bar{s}$. Moreover, given \bar{s} , aggregate output and unemployment is adjusted as

$$\begin{aligned} Y &= z \cdot \Lambda(\lambda) \cdot m(e(\bar{s}) \cdot K, L) \\ u &= 1 - \Omega(\lambda) \cdot m\left(\frac{e(\bar{s}) \cdot K}{L}, 1\right), \end{aligned}$$

and the aggregate matching efficiency is $\hat{\eta} = \Omega(\lambda) \cdot \eta \cdot e(\bar{s})$.

It remains for us to characterize the choice of recruiting effort s . First, given

³⁵The recent work by Mukoyama, Patterson and Sahin (2013) complements to Davis, Faberman and Haltiwanger (2012) by focusing on the job search intensity of worker side. Our paper focuses on the endogenous search effort by the firm side.

³⁶I also tried an alternative setup to endogenize firm's recruiting effort. It is the entrepreneurs who incur non-pecuniary disutility for recruiting effort. The alternative extension is available upon request.

(s, \bar{s}) , the decision by active entrepreneur- (a, φ) is formulated as below.

$$\pi(\varphi, s) \equiv \max_{s.t. \ p(\theta(\varphi), s) \cdot w(\varphi) = W} \{q(\theta(\varphi), s) \cdot (\varphi - w(\varphi))\}$$

The modified market tightness $\theta(\varphi)$ is pinned down by the FOC $\frac{\partial m(\theta(\varphi), 1)}{\partial \theta(\varphi)} = \frac{W}{\varphi}$. Second, given s , the individual decision rule on lending or borrowing depends on $\hat{\varphi}(s)$, where $\pi(\hat{\varphi}(s), s) = r$. Therefore s is determined by

$$\max \{ (1 - \sigma) \cdot \{ \lambda [1 - F(\hat{\varphi}(s))] \cdot \mathbb{E}[(\pi(\varphi, s) - r) | \varphi \geq \hat{\varphi}(s)] + r \} - c(s) \},$$

where $\lambda [1 - F(\hat{\varphi}(s))] \mathbb{E}[(\pi(\varphi, s) - r) | \varphi \geq \hat{\varphi}(s)] + r$ denotes the expected capital revenue with search effort s by workers, and σ proportion can be pledgeable to them, and $c(s)$ denotes the effort cost. I assume $e(0) = s_L > 0$. That is, if none of the entrepreneurs exert positive search effort, I am back to the baseline model. Besides, I assume $e'(s) > 0$, $e''(s) < 0$, $c(0) = 0$, $c'(s) > 0$, and $c''(s) \geq 0$. FOC upon the above equation delivers the endogenous choice of recruiting effort. First, the equilibrium search effort \bar{s} increases with (z, η, λ) . That is, these shocks will be amplified through the search effort. In particular, since $\hat{\eta} = \Omega(\lambda) \cdot \eta \cdot e(\bar{s}(z, \lambda, \eta))$, the decrease of either λ or η lowers aggregate matching efficiency in both direct and indirect way. I illustrate the amplification in Figure (1.23).

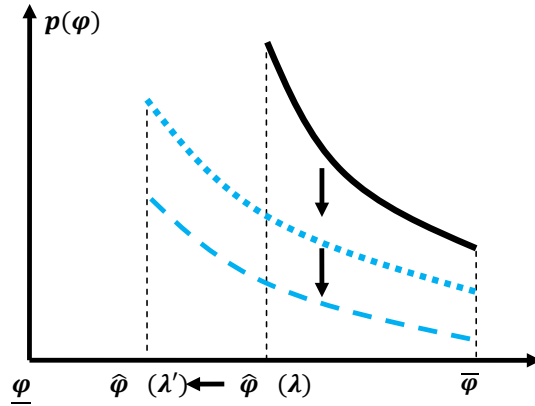


Figure 1.23: Unemployment Effect of Capital Reallocation with Endogenous Recruiting Effort

1.7.4 Appendix D - Proofs

Proof on Proposition 1

Proof. Substituting the participation constraint $p(\theta(\varphi))w(\varphi) = W$ into the objective function and using the fact that $p(\theta(\varphi)) = \frac{q(\theta(\varphi))}{\theta(\varphi)}$ yields

$$\pi(\varphi, W) = \max \{q(\theta(\varphi))\varphi - \theta(\varphi)W\},$$

and thus the FOC is $q'(\theta(\varphi)) = \frac{W}{\varphi}$, which pins down the market tightness $\theta(\varphi)$ in active submarket- $\varphi \in \Phi_A$. Using Implicit Function Theorem and the concavity of $q(\cdot)$ suggests that $\theta(\varphi)$ increases with φ and decreases with W . In turn, I recover the wage scheme as $w(\varphi) = \frac{W}{p(\theta(\varphi))}$. Since $p(\theta(\varphi))$ decreases with $\theta(\varphi)$, we know that $w(\varphi)$ increases with φ . Finally, using Envelope Theorem reveals that $\pi(\varphi, W)$ increases with φ and decreases with W . □

Proof on Lemma 1

Proof. The net revenue by entrepreneur- (a, φ) is

$$\max_{k \in [0, \lambda \cdot a]} \{\pi(\varphi, W) \cdot k - r \cdot (k - a) + (1 - \delta) \cdot a\},$$

where $\pi(\varphi, W) \cdot k$ denotes the capital revenue and $b = k - a$ is the debt if positive and the loan if negative. The above problem can be rewritten as $\max_{k \in [0, \lambda \cdot a]} [\pi(\varphi, W) - r] \cdot k + [r + (1 - \delta)] \cdot a$. Since the net revenue is linear k , and $k \in [0, \lambda \cdot a]$, only corner solutions, *i.e.*, $k = \lambda a$ or $k = 0$, will be considered. On the one hand, if $\pi(\varphi, W) > r$, the entrepreneurs not only want to engage in production, but also want to borrow as much as they can. On the other hand, if $\pi(\varphi, W) < r$, then the entrepreneurs prefer to lending to others. Since $\pi(\varphi, W)$ increases with φ , if I define the cut-off point $\widehat{\varphi}$ as $\pi(\widehat{\varphi}, W) = r$, then entrepreneurs choose to be active in production with a binding borrowing constraint if and only if $\varphi > \widehat{\varphi}$. □

Proof on Corollary 1

Proof. First, as shown in Lemma 1, the active set is $\Phi_A \equiv \{\varphi | \varphi \in [\widehat{\varphi}, \overline{\varphi}]\} = \{\varphi_i | \varphi_i \in [\widehat{\varphi}_i, \overline{\varphi}_i]\}$.

Therefore, the truncated distribution of productivity by active entrepreneurs is

$$F^A(\varphi) = \int_{\widehat{\varphi}}^{\varphi} f(\varphi|\varphi \geq \widehat{\varphi})d\varphi = \frac{F(\varphi) - F(\widehat{\varphi})}{1 - F(\widehat{\varphi})}.$$

Second, according to Proposition 1 and Lemma 1, wage scheme $w(\varphi)$ increases with φ and entrepreneurs with higher individual productivity, $q(\varphi)$, is more likely to be matched with workers. Therefore, the productivity distribution of finally matched capital is

$$F^M(\varphi) = \frac{\int_0^{\infty} \int_{\widehat{\varphi}}^{\varphi} k(\varphi', a) \cdot q(\varphi') \cdot h(\varphi, a) d\varphi da}{\int_{\widehat{\varphi}}^{\varphi} k(\varphi', a) \cdot q(\varphi') \cdot dF(\varphi')} = \frac{\int_{\widehat{\varphi}}^{\varphi} q(\varphi') \cdot dF(\varphi')}{\int_{\widehat{\varphi}}^{\varphi} q(\varphi') \cdot dF(\varphi')}.$$

Finally, I use the following lemma to prove $F^M(\varphi) < F^A(\varphi) < F(\varphi)$.

Lemma 3. Assume $\epsilon(\varphi) > 0$ for $\varphi \in [\underline{\varphi}, \overline{\varphi}]$. Given any $\widehat{\varphi} \in [\underline{\varphi}, \overline{\varphi}]$, define

$$F_1(\varphi) \equiv \frac{\int_{\widehat{\varphi}}^{\varphi} \epsilon(\varphi') d\varphi'}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi') d\varphi'}, \quad F_2(\varphi) \equiv \frac{\int_{\widehat{\varphi}}^{\varphi} \epsilon(\varphi') \vartheta(\varphi') d\varphi'}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi') \vartheta(\varphi') d\varphi'},$$

where $\vartheta(\varphi)$ increases with φ and is bounded by $[0, 1]$. Then $F_1(\varphi) \leq F_2(\varphi)$.

I leave the proof of this lemma at the end of this part. Now use this lemma to prove $F^M(\varphi) < F^A(\varphi) < F(\varphi)$. First, we can rewrite $F^A(\varphi)$ and $F^M(\varphi)$ as below.

$$F^A(\varphi) = \frac{\int_{\widehat{\varphi}}^{\varphi} f(\varphi') d\varphi'}{\int_{\widehat{\varphi}}^{\overline{\varphi}} f(\varphi') d\varphi'}, \quad F^M(\varphi) = \frac{\int_{\widehat{\varphi}}^{\varphi} f(\varphi') q(\varphi') d\varphi'}{\int_{\widehat{\varphi}}^{\overline{\varphi}} f(\varphi') q(\varphi') d\varphi'}.$$

Therefore, if we treat $\epsilon(\varphi)$ as $f(\varphi)$, and $\vartheta(\varphi)$ as $q(\varphi)$, which has been proved to increase with φ in Proposition 1, then using the above lemma immediately suggests $F^M(\varphi) \leq F^A(\varphi)$, *i.e.*, $F^M(\varphi)$ first-order stochastic dominates (FOSD) $F^A(\varphi)$. Moreover, we can rewrite $F^A(\varphi)$ as below.

$$F^A(\varphi) = \frac{\int_{\underline{\varphi}}^{\varphi} f(\varphi') \cdot \mathbf{1}_{\{\varphi' \geq \widehat{\varphi}\}} d\varphi'}{\int_{\underline{\varphi}}^{\overline{\varphi}} f(\varphi') \cdot \mathbf{1}_{\{\varphi' \geq \widehat{\varphi}\}} d\varphi'}.$$

If we treat $\vartheta(\varphi)$ as $\mathbf{1}_{\{\varphi \geq \widehat{\varphi}\}}$, which increases with φ and bounded by $[0, 1]$ in this scenario, then immediately the above lemma implies $F^A(\varphi) \leq F(\varphi)$. I close this part by proving the aforementioned lemma. Define $F_3(\varphi) \equiv F_2(\varphi) - F_1(\varphi)$. Then we have

$$\begin{aligned} F_3'(\varphi) &= F_2'(\varphi) - F_1'(\varphi) \\ &= \frac{\epsilon(\varphi) \vartheta(\varphi)}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi') \vartheta(\varphi') d\varphi'} - \frac{\epsilon(\varphi)}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi') d\varphi'} \\ &= \epsilon(\varphi) \frac{\left[\vartheta(\varphi) \int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi') d\varphi' - \int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi') \vartheta(\varphi') d\varphi' \right] \cdot \epsilon(\varphi)}{\left[\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi') \vartheta(\varphi') d\varphi' \right] \cdot \left[\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi') d\varphi' \right]}. \end{aligned}$$

Now I define $F_4(\varphi) \equiv \vartheta(\varphi) \int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi') d\varphi' - \int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi') \vartheta(\varphi') d\varphi'$. Then we immediately

know that, since $\vartheta(\varphi)$ is an increasing function in φ , so is $F_4(\varphi)$. Moreover, notice that $F_4(\widehat{\varphi}) < 0$ and $F_4(\overline{\varphi}) > 0$, and thus there exists a cut-off $\check{\varphi} \in (\widehat{\varphi}, \overline{\varphi})$ such that $F_4(\varphi) < 0$ when $\varphi \in (\widehat{\varphi}, \check{\varphi})$ and $F_4(\varphi) > 0$ when $\varphi \in (\check{\varphi}, \overline{\varphi})$. In turn, we know that,

$$F'_3(\varphi) \begin{cases} < 0 & \text{if } \varphi \in (\widehat{\varphi}, \check{\varphi}) \\ > 0 & \text{if } \varphi \in (\check{\varphi}, \overline{\varphi}). \end{cases}$$

Besides, since $F_3(\overline{\varphi}) = F_3(\widehat{\varphi}) = 0$, we know that $F_3(\varphi) \equiv F_2(\varphi) - F_1(\varphi) \leq 0$ is always satisfied.

□

Proof on Corollary 2

Proof. First, using the result on capital demand mentioned above, the constrained optimization by entrepreneur- (a, φ) can be rewritten as

$$V(a, \varphi; X) = \max \{ \log(c) + \beta \cdot \mathbb{E}[V(a', \varphi'; X') | X] \},$$

subject to $c + a' = \Psi(\varphi) \cdot a$, where $\Psi(\varphi) = \max \{ \pi(\varphi) - r, 0 \} \cdot \lambda + [r + (1 - \delta)]$. Then I substitute the capital demand of Lemma 1 into the budget constraint and thus reach the simplified version of the constrained optimization problem by entrepreneur- (a, φ) .

Second, I address the policy function. Guess the value function is linear with own net worth, *i.e.*, $V(a, \varphi) = C(\varphi) + D \cdot \log(a)$, then we have

$$V(a, \varphi) = C(\varphi) + D \cdot \log(a) = \max_{a' \in (0, \Psi(\varphi) \cdot a)} \{ \log(\Psi(\varphi) \cdot a - a') + \beta \cdot \mathbb{E}[C(\varphi') + D \cdot \log(a') | \varphi] \},$$

where $\Psi(\varphi) = \max \{ \pi(\varphi) - r, 0 \} \cdot \lambda + [r + (1 - \delta)]$. FOC suggests $a' = \left(\frac{\beta D}{1 + \beta D} \right) \cdot \Psi(\varphi) \cdot a$. In turn, the above Bellman equation can be rewritten as

$$C(\varphi) + D \cdot \log(a) = \log \left[\left(\frac{1}{1 + \beta D} \right) \cdot \Psi(\varphi) \cdot a \right] + \beta \cdot \left\{ \mathbb{E}[C(\varphi') | \varphi] + D \cdot \log \left[\left(\frac{\beta D}{1 + \beta D} \right) \cdot \Psi(\varphi) \cdot a \right] \right\}.$$

Therefore $D = \frac{1}{1 - \beta}$, and thus $a' = \beta \cdot \Psi(\varphi) \cdot a$. In turn, $d = \Psi(\varphi) \cdot a - a' = (1 - \beta) \cdot \Psi(\varphi) \cdot a$.

□

Proof on Lemma 2

Proof. Denote Φ^* as the efficient set of active capital and labor, *i.e.*, $\Phi^* = \{ \varphi | l(\varphi) > 0, v(\varphi) > 0 \}$. Assume the measure of Φ^* contains at least two types of productivity φ , then for $\varphi_i \in \Phi^*$,

the FOC suggests

$$\begin{aligned}\varphi_i \cdot m_v(v(\varphi_i), l(\varphi_i)) &= \mu_K \\ \varphi_i \cdot m_l(v(\varphi_i), l(\varphi_i)) &= \mu_L,\end{aligned}$$

where μ_K and μ_L denotes the Lagrangian multiplier of the constraints on capital and labor respectively. Then we have

$$\frac{m_v(v(\varphi_i), l(\varphi_i))}{m_l(v(\varphi_i), l(\varphi_i))} = \frac{m_v(1, \theta(\varphi_i))}{m_l(1, \theta(\varphi_i))} = \frac{\mu_K}{\mu_L},$$

where the first equation uses the fact that m_v and m_l are homogeneous of degree one. Immediately we know $\theta(\varphi_i)$ is constant for $\varphi_i \in \Phi^*$. Then we know that

$$\varphi_i \cdot m_v(\theta(\varphi_i)) = \mu_K.$$

Therefore φ_i is unique and is determined by μ_K and μ_L . Thus there is only one element in Φ^* . It then goes without say that $\Phi^* = \{\bar{\varphi}\}$. In turn,

$$Y^* = \int_{\Phi} \varphi m(v^*(\varphi), l^*(\varphi)) d\varphi = \bar{\varphi} \cdot m(K, L).$$

□

Proof on Proposition 2

Proof. First, the clearing condition could be further simplified as $\lambda \cdot [1 - F(\hat{\varphi})] = 1$. Using Implicit Function Theorem immediately suggests that $\hat{\varphi}$ increases with λ , and $\lim_{\lambda \rightarrow 1} \hat{\varphi} = \underline{\varphi}$ and $\lim_{\lambda \rightarrow \infty} \hat{\varphi} = \bar{\varphi}$.

Second, The aggregate output is defined as $Y = \int_0^\infty \int_{\underline{\varphi}}^{\bar{\varphi}} \varphi v(\varphi, a) q(\varphi) d\varphi da$. Since $v(\varphi, a) = k(\varphi, a)h(\varphi, a) = \lambda a f(\varphi)g(a) \cdot 1_{\varphi \in \Phi^A}$, the output can be rewritten as

$$Y = \left[\int_{\hat{\varphi}}^{\bar{\varphi}} \varphi \cdot q(\varphi) \cdot dF(\varphi) \right] \cdot \lambda K.$$

When the matching function in sub-labor market φ is $m(l(\varphi), v(\varphi)) = \eta \cdot l(\varphi)^{1-\gamma} v(\varphi)^\gamma$, then the matching probability by entrepreneur- φ is $q(\varphi) = \frac{m(l(\varphi), v(\varphi))}{v(\varphi)} = \eta \cdot \theta(\varphi)^{1-\gamma}$. In turn, the FOC is simplified as $q'(\varphi) = \eta \cdot (1 - \gamma) \cdot \theta(\varphi)^{-\gamma} = \frac{W}{\varphi}$, and thus $\theta(\varphi, W) =$

$\left[\frac{\eta(1-\gamma)\varphi}{W} \right]^{\frac{1}{\gamma}}$. As a result, we have

$$\begin{aligned} q(\varphi, W) &= \eta \cdot \left[\frac{\eta(1-\gamma)}{W} \right]^{\frac{1-\gamma}{\gamma}} \cdot \varphi^{\frac{1-\gamma}{\gamma}} \\ p(\varphi, W) &= \frac{W}{(1-\gamma) \cdot \varphi} \\ \pi(\varphi, W) &= \eta^{\frac{1}{\gamma}} \gamma (1-\gamma)^{\frac{1}{\gamma}-1} W^{1-\frac{1}{\gamma}} \cdot \varphi^{\frac{1}{\gamma}} \end{aligned}$$

and thus, for $\varphi \in \Phi_A$, the optimal wage scheme is as $w(\varphi, W) = \frac{W}{p(\varphi, W)} = (1-\gamma) \cdot \varphi$. Moreover, the labor resource constraint can be rewritten as

$$\lambda K \int_{\widehat{\varphi}}^{\overline{\varphi}} \theta(\varphi, W) dF(\varphi) = \left[\frac{\eta(1-\gamma)}{W} \right]^{\frac{1}{\gamma}} \cdot K \cdot \left[\lambda \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi) \right] = L.$$

Since $\lambda \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi) = \lambda [1 - F(\widehat{\varphi})] \cdot \left(\frac{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi)}{1 - F(\widehat{\varphi})} \right) = \mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right)$, we have

$$\left[\frac{\eta(1-\gamma)}{W} \right]^{\frac{1}{\gamma}} = \frac{L}{K \cdot \mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right)}.$$

Therefore the aggregate output can be further rewritten.

$$\begin{aligned} Y &= \int \int_{\widehat{\varphi}}^{\varphi_{max}} z \cdot \varphi v(\varphi) q(\varphi) d\varphi dG(a) \\ &= z \eta \cdot \left[\frac{\eta(1-\gamma)}{W} \right]^{\frac{1-\gamma}{\gamma}} \cdot K \left[\lambda \int_{\widehat{\varphi}}^{\varphi_{max}} \varphi^{\frac{1}{\gamma}} dF(\varphi) \right] \\ &= \left\{ z \eta \left(\mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right) \right)^\gamma \right\} \cdot K^\gamma L^{1-\gamma} \\ &= z \cdot \left(\mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right) \right)^\gamma \cdot m(K, L). \end{aligned}$$

which can be immediately verified by using change of variables. Then equilibrium TFP is obtained as below.

$$TFP \equiv \frac{Y}{K^\gamma L^{1-\gamma}} = z \cdot \eta \cdot \left(\mathbb{E} \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right) \right)^\gamma.$$

Now I characterize unemployment. By definition, the total matched labor (and capital) can be formulated as below.

$$N \equiv \int_0^\infty \int_{\widehat{\varphi}}^{\overline{\varphi}} v(\varphi, a) q(\varphi) d\varphi da = \left[\int_{\widehat{\varphi}}^{\overline{\varphi}} q(\varphi) \cdot dF(\varphi) \right] \cdot \lambda K = N = \mathbb{E}_F [q(\varphi) \mid \varphi \geq \widehat{\varphi}] \cdot K.$$

Moreover, I have

$$\begin{aligned}
N &\equiv \left[\int_{\widehat{\varphi}}^{\overline{\varphi}} q(\varphi) \cdot dF(\varphi) \right] \cdot \lambda K = \left[\eta \int_{\widehat{\varphi}}^{\overline{\varphi}} \left[\frac{\eta(1-\gamma)\varphi}{W} \right]^{\frac{1-\gamma}{\gamma}} \cdot dF(\varphi) \right] \cdot \lambda K \\
&= \eta \left(\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} \cdot dF(\varphi) \right) \left[\frac{L}{K \cdot \mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right)} \right]^{1-\gamma} \lambda K = \Omega(\lambda) \cdot m(K, L),
\end{aligned}$$

where $\Omega(\lambda) \equiv \frac{\mathbb{E}_F \left(\varphi^{\frac{1-\gamma}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right)}{\left[\mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right) \right]^{1-\gamma}}$. Since $\gamma \in (0, 1)$, Jensen's inequality suggests $\mathbb{E}_F \left(\varphi^{\frac{1-\gamma}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right) < \left[\mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right) \right]^{1-\gamma}$ and thus $N < m(K, L) = N^*$. Consequently, unemployment is

$$u \equiv 1 - \frac{N}{L} = 1 - \left\{ \frac{\mathbb{E}_F \left(\varphi^{\frac{1-\gamma}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right)}{\left[\mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right) \right]^{1-\gamma}} \right\} \cdot \eta \cdot \left(\frac{K}{L} \right)^\gamma.$$

Therefore, u decreases with η , but has nothing to do with z .

Now I address the factor price in labor and credit markets in turn. On the one hand, $\left[\frac{\eta(1-\gamma)}{W} \right]^{\frac{1}{\gamma}} = \frac{L}{K \cdot \mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right)}$, we know that

$$W = \eta(1-\gamma) \left[\left(\frac{K}{L} \right) \cdot \mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right) \right]^\gamma.$$

On the other hand, I already prove that

$$Y = \left\{ \eta \left(\mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right) \right)^\gamma \right\} \cdot K^\gamma L^{1-\gamma}.$$

Immediately we have $W = \frac{\partial Y}{\partial L}$. Then I need to pin down the interest rate in the credit market. Since $\pi(\widehat{\varphi}, W) = r$ and

$$\pi(\varphi, W) = q(\varphi, W)(\varphi - w(\varphi)) = \eta(\theta(\varphi, W))^{1-\gamma} \gamma \varphi,$$

the market tightness can be rewritten as $\theta(\varphi, W) = \left(\frac{r}{\eta \gamma \widehat{\varphi}} \right)^{\frac{1}{1-\gamma}} \cdot \left(\frac{\varphi}{\widehat{\varphi}} \right)^{\frac{1}{\gamma}}$. In turn, the labor resource constraint can be re-formulated as

$$\lambda K \left(\frac{r}{\eta \gamma \widehat{\varphi}} \right)^{\frac{1}{1-\gamma}} \cdot \int_{\widehat{\varphi}}^{\overline{\varphi}} \left(\frac{\varphi}{\widehat{\varphi}} \right)^{\frac{1}{\gamma}} dF(\varphi) = L.$$

Therefore, the interest rate can be rewritten as

$$\frac{r}{\partial Y / \partial K} = \frac{\widehat{\varphi}^{\frac{1}{\gamma}}}{\mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right)} = \frac{\widehat{\varphi}_i^{\frac{1}{\gamma}}}{\mathbb{E}_{F^i} \left(\varphi_i^{\frac{1}{\gamma}} \mid \varphi_i \in [\widehat{\varphi}_i, \overline{\varphi}_i] \right)} \equiv 1 - \tau_K.$$

Since $\mathbb{E}_F \left[\left(\frac{\varphi}{\widehat{\varphi}} \right)^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right]$ decreases $\widehat{\varphi}$ and $\widehat{\varphi}$ increases with λ , we know that $\frac{r}{\partial Y / \partial K} = \frac{1}{\mathbb{E}_F \left[\left(\frac{\varphi}{\widehat{\varphi}} \right)^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right]}$, which increases with λ . Moreover, we have

$$\lim_{\lambda \rightarrow \infty} \frac{r}{\partial Y / \partial K} = \lim_{\alpha \rightarrow 0} \frac{r}{\partial Y / \partial K} = 1, \quad \lim_{\overline{\varphi}_i \rightarrow \infty} \frac{r}{\partial Y / \partial K} = 1 - \frac{\alpha}{\gamma}.$$

Finally, I show why $\Omega(\lambda)$ increases with λ . It is immediately done by using Assumption 1. Furthermore, I make further characterization on $\Omega(\lambda)$. Notice that we can rewrite $\Omega(\lambda)$ as $\Omega(\lambda) = \frac{\lambda^\gamma \cdot \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)}{\left[\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi) \right]^{1-\gamma}}$. Denote $A(\lambda) = \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)$ and $B(\lambda) = \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi)$, then after some algebraic manipulation, we have

$$\Omega'(\lambda) = \frac{\lambda^\gamma}{B(\lambda)^{1-\gamma}} \left\{ \left(\frac{\gamma}{\lambda} \right) A(\lambda) + A'(\lambda) \left[1 - (1-\gamma) \left(\frac{A(\lambda)}{B(\lambda)} \right) \widehat{\varphi}(\lambda) \right] \right\},$$

where

$$A'(\lambda) = -(\widehat{\varphi}(\lambda))^{\frac{1-\gamma}{\gamma}} \cdot f(\widehat{\varphi}(\lambda)) \cdot \widehat{\varphi}'(\lambda) = -(\widehat{\varphi}(\lambda))^{\frac{1-\gamma}{\gamma}} \cdot f(\widehat{\varphi}(\lambda)) \cdot \left[\frac{1}{\lambda^2} \cdot \frac{1}{f(\widehat{\varphi})} \right] = -(\widehat{\varphi}(\lambda))^{\frac{1-\gamma}{\gamma}} \cdot \left(\frac{1}{\lambda^2} \right).$$

Therefore, $\Omega'(\lambda) > 0$ if and only if

$$\left(\frac{\gamma}{\lambda} \right) A(\lambda) + A'(\lambda) \left[1 - (1-\gamma) \left(\frac{A(\lambda)}{B(\lambda)} \right) \widehat{\varphi}(\lambda) \right] = \left(\frac{1}{\lambda} \right) \left\{ \gamma \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi) - \left(\frac{1}{\lambda} \right) (\widehat{\varphi}(\lambda))^{\frac{1-\gamma}{\gamma}} \left[1 - (1-\gamma) \left(\frac{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi)} \right) \widehat{\varphi}(\lambda) \right] \right\} > 0,$$

or, equivalently, $\Omega'(\lambda) > 0$ if and only if

$$\gamma > \left(\frac{1}{\lambda} \right) \cdot \left[\frac{(\widehat{\varphi}(\lambda))^{\frac{1-\gamma}{\gamma}}}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)} \right] \cdot \left[1 - (1-\gamma) \left(\frac{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi)} \right) \widehat{\varphi}(\lambda) \right].$$

Notice that

$$\begin{aligned}
\left(\frac{1}{\lambda}\right) \cdot \left[\frac{(\widehat{\varphi}(\lambda))^{\frac{1-\gamma}{\gamma}}}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)} \right] \cdot \left[1 - (1-\gamma) \left(\frac{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi)} \right) \widehat{\varphi}(\lambda) \right] &= \left(\frac{1}{\lambda}\right) \cdot \left\{ \frac{(\widehat{\varphi}(\lambda))^{\frac{1-\gamma}{\gamma}}}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)} - (1-\gamma) \cdot \frac{(\widehat{\varphi}(\lambda))^{\frac{1}{\gamma}}}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi)} \right\} \\
&= \left(\frac{1}{\lambda}\right) \cdot \left\{ \frac{1}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \left(\frac{\varphi}{\widehat{\varphi}(\lambda)}\right)^{\frac{1-\gamma}{\gamma}} dF(\varphi)} - \frac{1-\gamma}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \left(\frac{\varphi}{\widehat{\varphi}(\lambda)}\right)^{\frac{1}{\gamma}} dF(\varphi)} \right\}.
\end{aligned}$$

Therefore $\Omega'(\lambda) > 0$, or Assumption 1 is held, if and only if

$$\mathbb{E}_F \left[\left(\frac{\varphi}{\widetilde{\varphi}} \right)^{\frac{1}{\gamma}} \mid \varphi \in (\widetilde{\varphi}, \overline{\varphi}) \right] \cdot \left\{ 1 - \left(\frac{1}{\gamma} \right) \cdot \left[\frac{1 - F(\widetilde{\varphi})}{\widetilde{\varphi} \cdot f(\widetilde{\varphi})} \right] \right\} \leq 1.$$

□

Proof on Corollary 3

Proof. There are at least two ways to prove this result. On the one hand, we can verify this claim by the following reasoning. Since I have proved that the optimal wage scheme in active sub-labor markets is $w(\varphi) = (1-\gamma)\varphi$, we know that all active entrepreneurs get γ proportion of realized output. Then by definition, the aggregate accumulated wealth by entrepreneurs, $\{\int \Psi(\varphi) dF(\varphi)\} \cdot K$, should equal to the capital stock after depreciation, $(1-\delta) \cdot K$, plus γ proportion of the aggregate output, γY . On the other hand, we can prove the result by straightforward calculation as below. As defined in the context, $\Psi(\varphi) = \lambda \cdot \max\{\pi(\varphi) - r, 0\} + r + (1-\delta)$. Then we have

$$\begin{aligned}
\int \Psi(\varphi) dF(\varphi) &= \int [\lambda \cdot \max\{\pi(\varphi) - r, 0\} + r + (1-\delta)] dF(\varphi) \\
&= \lambda \cdot \int \max\{\pi(\varphi) - r, 0\} dF(\varphi) + r + (1-\delta) \\
&= \left\{ \lambda \cdot \int_{\widehat{\varphi}}^{\overline{\varphi}} \left[\left(\frac{\varphi}{\widehat{\varphi}} \right)^{\frac{1}{\gamma}} - 1 \right] + 1 \right\} \cdot r + (1-\delta).
\end{aligned}$$

Then using the clearing condition in credit market, *i.e.*, $\lambda = 1/[1 - F(\widehat{\varphi})]$, then we have

$$\begin{aligned}
\left\{ \int \Psi(\varphi) dF(\varphi) \right\} \cdot K &= \left\{ \lambda \cdot \int_{\widehat{\varphi}}^{\overline{\varphi}} \left[\left(\frac{\varphi}{\widehat{\varphi}} \right)^{\frac{1}{\gamma}} - 1 \right] + 1 \right\} \cdot rK + (1-\delta)K \\
&= \left\{ \mathbb{E}_F \left[\left(\frac{\varphi}{\widehat{\varphi}} \right)^{\frac{1}{\gamma}} - 1 \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right] + 1 \right\} \cdot (1-\tau) \cdot \left(\frac{\partial Y}{\partial K} \cdot K \right) + (1-\delta)K \\
&= \gamma Y + (1-\delta)K.
\end{aligned}$$

Therefore the aggregate transition dynamics is obtained as below.

$$K_{t+1} = \beta \cdot \left[\int_{\Phi} \Psi_t(\varphi) dF_t(\varphi) \right] \cdot K_t = \beta \cdot [\gamma Y_t + (1-\delta) K_t].$$

□

Proof on Proposition 3

Proof. Given w , the active entrepreneur's decision is

$$\pi(\varphi) \cdot k = \max_l \{ \eta \varphi k^\gamma l^{1-\gamma} - wl \}.$$

FOC suggests $l = (\eta \varphi)^{\frac{1}{\gamma}} \left(\frac{1-\alpha}{w} \right)^{\frac{1}{\gamma}} k$, which in turn implies

$$\pi(\varphi) = \gamma (\eta \varphi)^{\frac{1}{\gamma}} \left(\frac{1-\alpha}{w} \right)^{\frac{1-\gamma}{\gamma}}.$$

Since $\pi(\varphi)$ increases with φ , the cut-off point $\widehat{\varphi}$ is determined by $\pi(\widehat{\varphi}, w) = r$. The capital and labor demand then is obtained as below.

$$\begin{aligned} k(\varphi, a) &= \begin{cases} \lambda \cdot a & \text{if } \varphi \geq \widehat{\varphi} \\ 0 & \text{if } \varphi < \widehat{\varphi} \end{cases} \\ l(\varphi, a) &= (\eta \varphi)^{\frac{1}{\gamma}} \left(\frac{1-\alpha}{w} \right)^{\frac{1}{\gamma}} k(\varphi, a). \end{aligned}$$

In turn, the clearing conditions in credit market, $\int_0^\infty \int_{\widehat{\varphi}}^{\overline{\varphi}} k(\varphi, a) h(\varphi, a) d\varphi da = K$, can be simplified as $\lambda \cdot [1 - F(\widehat{\varphi})] = 1$. Meanwhile, the resource constraint in the labor market, $\int_0^\infty \int_{\widehat{\varphi}}^{\overline{\varphi}} l(\varphi, a) h(\varphi, a) d\varphi da = L$, can be rewritten as below.

$$\left[\frac{\eta(1-\alpha)}{w} \right]^{\frac{1}{\gamma}} \left(\lambda K \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi) \right) = L.$$

Since $\lambda \cdot [1 - F(\widehat{\varphi})] = 1$, we have $\lambda \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi) = \mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right)$, and therefore $\left[\frac{\eta(1-\alpha)}{w} \right]^{\frac{1}{\gamma}} = \frac{L}{K \cdot \mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right)}$.

By definition, aggregate output is

$$\begin{aligned} Y &= \int_0^\infty \int_{\widehat{\varphi}}^{\overline{\varphi}} y(\varphi, a) h(\varphi, a) d\varphi da \\ &= \int_0^\infty \int_{\widehat{\varphi}}^{\overline{\varphi}} \left(\frac{\pi(\varphi, w) k(\varphi, a)}{\gamma} \right) h(\varphi, a) d\varphi da \\ &= \left[\frac{\eta(1-\alpha)}{w} \right]^{\frac{1-\gamma}{\gamma}} \left(\lambda K \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi) \right) \\ &= \left\{ \eta \left(\mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right) \right)^\gamma \right\} \cdot K^\gamma L^{1-\gamma}, \end{aligned}$$

and thus $TFP \equiv \frac{Y}{K^\gamma L^{1-\gamma}} = \eta z \left(\mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right) \right)^\gamma$.

□

Proof on Corollary 4

Proof. By definition, we have

$$\begin{aligned}
l_t - l_{t+1} &= k_t(\varphi_t, a_t)q_t(\varphi_t) - \mathbb{E}[k_{t+1}(\varphi_{t+1}, a_{t+1}) \cdot q_{t+1}(\varphi_{t+1})] \\
&= \lambda_t a_t \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_t\}} q_t(\varphi_t) - \mathbb{E}_{\varphi_{t+1}} \left[\lambda_{t+1} a_{t+1} \cdot \mathbf{1}_{\{\varphi_{t+1} \geq \widehat{\varphi}_{t+1}\}} q_{t+1}(\varphi_{t+1}) \right] \\
&= \lambda_t a_t \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_t\}} q_t(\varphi_t) - \lambda_{t+1} a_{t+1} \left[\rho \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_{t+1}\}} q_{t+1}(\varphi_t) + (1 - \rho) \cdot \int_{\Phi} \mathbf{1}_{\{\varphi \geq \widehat{\varphi}_{t+1}\}} q_{t+1}(\varphi) \cdot dF(\varphi) \right] \\
&= a_t \cdot \left\{ \lambda_t \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_t\}} q_t(\varphi_t) - \lambda_{t+1} \beta \Psi(\varphi_t) \cdot \left[\rho \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_t\}} q_{t+1}(\varphi_t) + (1 - \rho) \cdot \int_{\Phi} \mathbf{1}_{\{\varphi \geq \widehat{\varphi}_{t+1}\}} q_{t+1}(\varphi) \cdot dF(\varphi) \right] \right\} \\
&\equiv a_t \cdot \Delta_{t,t+1}(\varphi^t).
\end{aligned}$$

□

Proof on Corollary 5

Proof. The first part is obvious. When it comes to growth rate, by definition and using the results of the first part, we have

$$\begin{aligned}
\mathbb{E} \left[\frac{k_{t+1}}{k_t} | (k_t, \varphi_t; X_t) \right] &= \beta \cdot \left(\frac{\Psi_t(\varphi_t)}{\lambda_t} \right) \cdot \mathbb{E} \left[\mathbf{1}_{\{\varphi' \geq \widehat{\varphi}(X')\}} \cdot \lambda_{t+1} | (\varphi_t, X_t) \right] \\
&= \beta \cdot \left(\frac{\Psi_t(\varphi_t)}{\lambda_t} \right) \cdot \mathbb{E} \left\{ \left[\rho \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_{t+1}\}} + (1 - \rho) \cdot (1 - F(\widehat{\varphi}_{t+1})) \right] \cdot \lambda_{t+1} | (\varphi_t, X_t) \right\}.
\end{aligned}$$

Meanwhile,

$$\begin{aligned}
\mathbb{E} \left[\frac{n_{t+1}}{n_t} | (n_t, \varphi_t; X_t) \right] &= \frac{\mathbb{E} [k_t(\varphi'_{t+1}, a_{t+1}) q_{t+1}(\varphi'_{t+1})]}{k_t(\varphi_t, a_t) q_t(\varphi_t)} \\
&= \beta \cdot \Psi_t(\varphi_t) \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left[\rho \cdot \left(\frac{q_{t+1}(\varphi_t)}{q_t(\varphi_t)} \right) + (1 - \rho) \cdot \left(\frac{\int_{\widehat{\varphi}_{t+1}}^{\overline{\varphi}} q(\varphi_{t+1}) dF(\varphi_{t+1})}{q_t(\varphi_t)} \right) \right].
\end{aligned}$$

□

Chapter 2

Asset Exchange with Search Frictions and Costly Information Acquisition

2.1 Introduction

A large number of financial assets, such as derivatives, futures, swaps, corporate bonds and equity, are traded in both decentralized and centralized markets.¹ A decentralized market (DM), for example an over-the-counter market, is mainly characterized by search frictions and bilateral trade; see Duffie (2012). A centralized market (CM), for example the New York stock exchange, has terms of trade publicly displayed and search frictions are not important. Motivated by these phenomena on co-existence of markets with quite different structures, we raise the following questions. Since trading parties could enjoy a publicly displayed price without search frictions in CM, why do some agents bother to trade in DM? When could CM and DM co-exist for asset trading? What is the implication of market co-existence for asset liquidity?²

This paper develops a tractable model to address the above positive and normative questions, in particular conditions under which centralized and decentralized

¹See Harris (2003) and Bolton, Santos and Scheinkman (2011b).

²Market liquidity, as emphasized by Chang (2012), consists of two-dimensional measurement. One is on the price while the other one is on the trading speed for trading assets in secondary market. Our paper takes into account both kind of indicators.

markets (CM/DM) co-exist for asset trading. Our paper is definitely not the first one to consider market co-existence for asset exchange.³ For theory, see Hall and Rust (2003), Miao (2006) and Bolton, Santos and Scheinkman (2011b) among others. For empirics, Biais and Green (2007) clearly document the secular migration of corporate-bond trading from CM to DM in the past century. Moreover, as shown in Harris (2003), equity trading has also recently become less centralized.

This paper adds value to the literature in two ways. First we illustrate a novel interaction of information frictions and search frictions and their roles in explaining endogenous market co-existence. Secondly, which illustrating the determinants of the migration of asset trading, the model lends insight into the heterogeneous welfare effect of a government asset purchase program.

In the benchmark case, *i.e.*, if there is no information asymmetry on asset payoffs, CM is shown to always dominate DM for asset trading in equilibrium. When asset payoffs and liquidity shock (discount factor, or called trading motive) are private information, there also exists a self-fulfilling equilibrium in which trading parties concentrate in CM.⁴⁵ A more intriguing case is whether the equilibrium with market co-existence can be supported. On the one hand, the seller's liquidity shock is always private information. On the other hand, buyers could always stay uninformed about asset payoffs. Then buyers could post a publicly displayed price in CM, at which demand equals supply. Alternatively, buyers can acquire costly information to avoid adverse selection in CM. The informed buyers could then propose a trading menu different from the unique price posted by uninformed buyers in CM. When sellers with high-quality assets self-select into the contracts offered by informed buyers, search frictions may emerge due to coordination failure. That is, information investment and search frictions are two aspects in DM.

Due to strategic complementarity between sellers and buyers on the choice of trading venues, there always exists an equilibrium in which only CM survives for asset exchange. Market co-existence is shown to be sustainable only when adverse selection in CM is severe, matching efficiency in DM is high, and the information cost is low enough. That is, we have multiple equilibria in latter case. To ease

³Our paper focus on the endogenous co-existence of centralized and decentralized market. Pagnotta and Phillipon (2011) also explore market co-existence, but they are engaged in the market fragmentation on trading in organized exchanges with different trading speed.

⁴In Section 2, we fully characterize three alternative cases, in which either of them is private information and both of them are private information.

⁵Sometimes asset payoffs and liquidity shock are named the common value and private value of assets respectively. See Chang (2012) for example.

the analysis of comparative statics, we always pick up the equilibrium with markets co-existence whenever it can be supported. Then we conclude that, as matching efficiency in DM increases and the information cost decreases, more trade migrates from CM with adverse selection to DM with search frictions. In the limit, DM with search frictions converges to CM with complete information.

Information investment serves as buyer's natural response to alleviate adverse selection. However, if we aggregate the revenues by all sellers, then immediately we know that information acquisition is purely a resource waste in our exchange economy.⁶ In addition to information investment, the unmatched trading in DM also contributes to the deadweight loss. However, since sellers are heterogeneous in their asset payoff and trading motives, closing DM is not Pareto improvement. Based on this observation, we move on to address the heterogeneous effect of a government asset purchase program, for example Troubled Asset Relief Program (TARP). We are particularly interested in the following question. If government is assumed to have access to a lower information cost, or a more efficient matching technology, will all sellers be better off with government intervention? We show that, when government steps in with a self-financing scheme, sellers with high-quality assets are better off while the others are worse off. Therefore, even though in our simple exchange model, which is free of incentive effect in production, a self-financing government asset purchase program does not necessarily makes everyone better off.

As emphasized by Levine (2005), liquid provision and resources reallocation in secondary market is one of the key functions of financial industry. Asset owners could enjoy market liquidity by transferring claims if secondary market functions well. However, adverse selection may dampen potential trade. To address information frictions, we introduce both adverse selection and costly state verification into our model. That is, in addition to posting a pooling price in CM, buyers could also choose to acquire costly information on asset payoffs. Then they can propose an optimal contract with bilateral trading in DM. Therefore, in addition to the literature on market co-existence, our paper is also related to the literature on the liquidity effect of information frictions. Earlier theory include Glosten and Milgrom (1985), Kyle (1985) and Williamson and Wright (1994) among others. Recent literature mainly consists of Eisfeldt (2004), Bolton, Santos and Scheinkman (2011a), Malherbe (2012), Kurlat (2012) and Tirole (2012). All of these papers assume a unique

⁶Buyers are assumed to be fully competitive and thus their gain is irrelevant for calculating the social welfare.

price in a competitive centralized market with adverse selection. Moreover, Guerrieri, Shimer and Wright (2010), Guerrieri and Shimer (2012a,b), Chang (2012), Chiu and Koepl (2011) address the effect of information asymmetry on asset trading with search frictions. Alternatively, Tirole and Farhi (2012) and Andolfatto, Berentsen and Waller (2013) adopt costly information acquisition to address the information asymmetry between a single seller and a single buyer. Similar to classic literature on security design, these two papers suggests information investment could be undue diligence under certain conditions.⁷

Our paper is most related to the independent works by Guerrieri and Shimer (2012b) and Chang (2012). All the three papers consider two-dimensional private information for asset trading, one is asset payoff while the other is trading motive.⁸ Guerrieri and Shimer (2012b) show when only asset quality is private information, there exists a unique separating equilibrium. Market illiquidity serves as the separating device. However, when both asset quality and the desire to sell are private information, the economy would be characterized by a unique partial pooling equilibrium. Chang (2012), which also considers private information on asset quality and trading motives with the framework of directed search, delivers similar conclusions. There are several key differences. First of all, our paper adopt different modeling strategy to address private information. Guerrieri and Shimer (2012b) use market illiquidity as a signaling device while we adopt costly information acquisition. In our model, CM is subject to adverse selection due to private information on asset's common and private values. Meanwhile, buyers could reduce information asymmetry from two to one dimension and then launch optimal contract to sellers self-selecting into DM. Secondly, the sub-markets with competitive search share very similar market structure in their papers. In contrast, our model offers a framework with endogenous co-existence of CM and DM, two kinds of markets with quite different characteristics.

The rest of this paper proceeds as follows. Section 2 sets up a stylized model and analyzes agent's choice of trading venues between CM and DM in partial equilibrium. Section 3 closes the model in general equilibrium. Section 4 uses the model to examine the heterogeneous effect of government asset purchase program. Section 5

⁷Security design is a burgeoning field with lots of interesting papers, say, DeMarzo and Duffie (1999), DeMarzo (2005) and Dang, Gorton and Holmström (2010) etc. Since our focus is not on security design, we would not give this field a fair treatment for the literature review. Instead, we only focus on information investment, which is also a key issue in this field.

⁸See Guerrieri and Shimer (2012)b for the comparison between their paper and Chang (2012).

concludes. The Appendix A pools proofs omitted in the context. The Appendix B documents several pieces of model extension.

2.2 Model

2.2.1 Environment

The economy is populated by two kinds of risk-neutral agents and lasts for two periods. First, there is unit measure of asset sellers. Each of them is endowed with one unit of indivisible Lucas tree at the beginning of $t = 1$. Seller's utility function is $U^S(x, \delta) = c_1 + \delta \cdot c_2$, where $c_1 \geq 0$ and $c_2 \geq 0$ denotes consumption at $t = 1$ and $t = 2$, and x and δ the idiosyncratic asset payoff and discount factor respectively. Therefore sellers are heterogeneous in both common value and private value. For notational ease, we label them as seller- (x, δ) . For simplicity, we assume these two distributions are independent of each other. On one hand, asset payoff is drawn from a continuous distribution $F(x)$ with support $[x_L, x_H]$. Discount factor, on the other hand, conforms to a distribution $G(\delta)$ with support $[0, 1]$. Trees only deliver consumption goods at $t = 2$. We assume sellers cannot produce. Therefore maturity mismatch may emerge if some sellers want to sell their trees to consume in $t = 1$. This is in particular true for sellers with $\delta = 0$.

Secondly, we assume there is a continuum of asset buyers. For simplicity, we assume no occupational choice between buyers and sellers ex ante.⁹ Buyers have access to a linear production technology with labor input at $t = 1$. There is no aggregate shock to this economy. However, we assume it is not feasible for sellers to issue contingent claims. Besides, no credit is assumed to be enforceable. Additionally, the limited commitment makes it impossible for sellers to signal in secondary market. Consequently, assets serve as medium of exchange, *i.e.*, sellers could transfer asset ownership to buyers for consumption at $t = 1$. In turn buyers would have to produce consumption goods to purchase the trees at $t = 1$, and consume the fruits at $t = 2$. Therefore when talking about liquidity, we exclusively mean market liquidity rather than funding liquidity.¹⁰

⁹Bolton et al (2011b) discusses the endogenous choice between financial service and real business.

¹⁰See Brunnermeier and Pedersen (2009) for the details on market and funding liquidity.

In addition to staying uninformed, buyers can also acquire costly information. More specifically, buyers could pay information cost κ with their labor disutility. Then they could perfectly detect the payoff of any asset. The action of information investment is publicly observed. Each information investment can *only* verify the quality of one-unit asset. We denote buyer's utility function as $U^B = -l_1^B - \kappa \cdot \mathbf{1}_{\{Info-invest\}} + \mathbb{E}(c_2^B)$, where l_1^B denotes the disutility from producing l_1^B units of goods, and c_2^B denotes the consumption goods at $t = 2$ from the trees the buyer purchase at $t = 1$.¹¹ Buyers are fully competitive so that they would earn zero profit from asset trading in equilibrium.

To fully characterize the expected revenue $\mathbb{E}(c_2^B)$, we need to specify the details on how assets are traded between sellers and buyers. On one hand, if certain buyer does not incur information cost, she would have no idea on the exact quality of assets. Thus she can only buy asset with a publicly displayed price p at which demand equals supply in equilibrium. On the other hand, if some buyer acquires costly information, she could follow uninformed buyers to post a publicly displayed price p . Alternatively, she could propose a trading menu for sellers *self-selecting* into the contract. Notice that informed buyers could detect the asset payoff x , but they still cannot directly observe δ , the discount factor of asset sellers. Without loss of generality, informed buyers uses direct mechanism $\{q(x, \delta), \tau(x, \delta)\}$. When sellers with asset payoff x report their type as δ , $q(x, \delta)$ is the probability that an asset transferred to buyers while $\tau(x, \delta)$ is the consumption paid to sellers.

Now it is time for us to be clear on our definition of centralized and decentralized markets (CM/DM). The former is a market where assets are traded at a publicly displayed price p . That is, sellers could always successfully sell their assets in CM at p without any search frictions. In contrast, as noted by Duffie (2012), DM is characterized by search and matching. That is, it takes time for sellers and buyers to find their trading partners. Since we assume each information investment can *only* verify the quality of one unit of asset, sellers and informed buyers will take bilateral trading. Therefore DM emerges in the bilateral trading since sellers may fail to coordinate with each other about which buyers to resort to. It is true whether it be random search or competitive (directed) search. We use random search in the

¹¹Some asset trading is dealer-intermediated in our real life, with corporate bonds just being a case. We assume away the intermediation in this paper. It contributes to great tractability for our focus on equilibrium choice. Or, a cheap interpretation is that we combine the roles of dealers and buyers and is exclusively engaged in the trading frictions due to private information on heterogeneity of seller side. The price of assuming away dealers in DM is that there is no room to use our model to discuss the bid-ask spread and other important dealer-related financial phenomenon.

baseline model and make robust check with competitive search in the Appendix B. We assume matching technology $m(b, s)$ in DM is exogenously given, increases with both augments, is homogeneous of degree one, $m(b, 0) = m(0, s) = 0$, and $m(b, s) \leq \min\{b, s\}$, where b and s denote the measure of buyers and sellers in DM.

Buyers and sellers move simultaneously. Buyers make their choice on information investment or not. Sellers decides whether and where to trade. For sellers who choose to go to DM but are not successfully matched, we assume they can no longer try CM and instead go directly to next period.¹² In our benchmark, we model liquidity shock in the simplest way as by Diamond and Dybvig (1983). We assume δ conforms to binomial distribution with $\delta \in \{0, 1\}$, $Pr\{\delta = 0\} = \pi \in (0, 1)$ and $Pr\{\delta = 1\} = 1 - \pi$.¹³ In the end, we use Figure 2.1 to summarize the time-line.

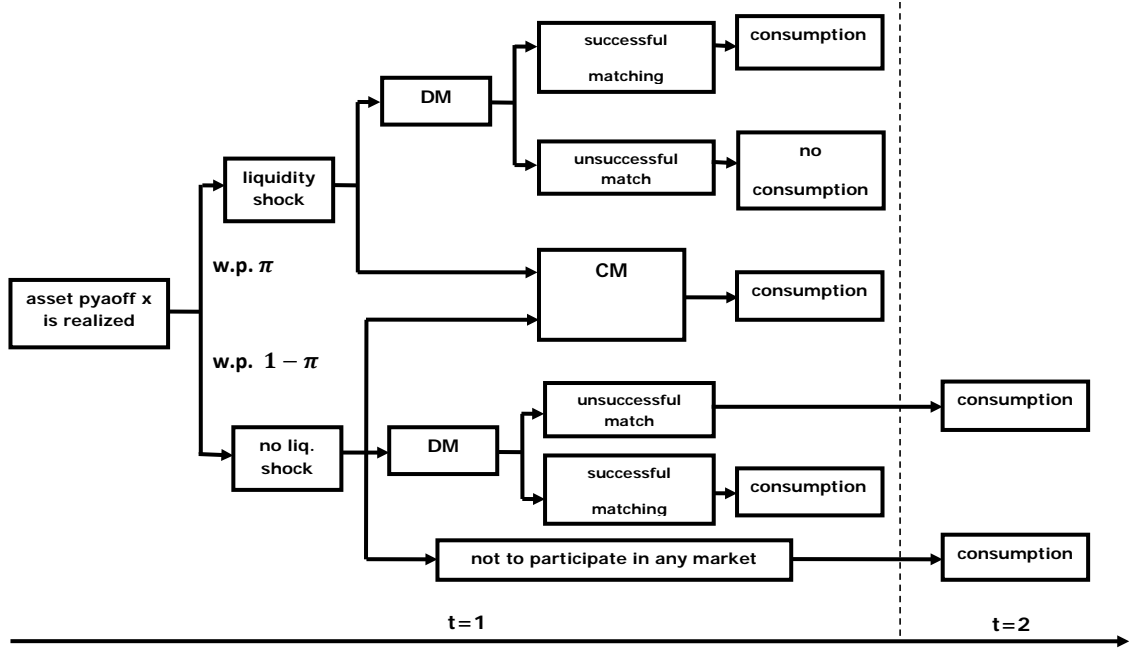


Figure 2.1: Raw Time-line with All Possible Paths

¹²We also consider an alternative scenario in which DM and CM are connected. That is, sellers have no commitment and are allowed to put their order at both markets. If sellers are not matched in DM, they still have the opportunity to liquidate their assets in CM if want to. Most of the qualitative conclusions in the context still hold.

¹³

We address in the Appendix B the general case in which δ is continuously distributed over $[0, 1]$.

2.2.2 Seller's Problem

As suggested above, buyers can incur fixed evaluation cost κ and then choose to go to the DM for asset trading. Then buyers would have no information disadvantage on x in DM. The terms of trade in DM for seller- (x, δ) is then determined by Nash bargaining under complete information. By complete information, we mean both asset payoff x and liquidity shock δ are publicly observable without any cost. It is worth noting that seller's outside option crucially depends on δ . Since we assume sellers cannot trade in CM and DM in the same time, even though two markets co-exist, the outside option of seller- (x, δ) going to DM is $x \cdot 1_{\{\delta=1\}}$. For sellers with $\delta = 0$, given

For those with $\delta = 0$, the terms of trade is determined by $\max_{\omega} \{\omega^{\eta} \cdot (x - \omega)^{1-\eta}\}$, which delivers $\omega(x) = \eta \cdot x$ for x . When bargaining with buyers in DM, seller- $(x, \delta = 0)$ and seller- $(x, \delta = 1)$ have different outside option. The outside option is zero and x for the former and latter respectively. Thus strictly speaking, the bargaining setting $\max_{\omega} \{\omega \cdot (x - \omega)^{1-\eta}\}$ is a reasonable for the former but not for the latter group of asset sellers. Fortunately, this subtle observation does not overthrow our analysis to come. Even though the “bargaining power” of seller with $\delta = 1$ could be higher than that of seller with $\delta = 0$, they would never try DM. This claim is immediately obtained by the following argument. Buyers in DM would charge at least something from the trading surplus. As a result, the best possible terms of trade for seller- $(x, \delta = 1)$ would always be strictly lower than x . For those sellers, they could always gain x by waiting until $t = 2$. Thus they would never try DM, even though the rule of splitting trading surplus for them is different from that for sellers with $\delta = 0$. Consequently, even though liquidity shock is always unobservable, buyers can infer it from seller's choice of trading venues.

For those with $\delta = 1$, there is no trading surplus in DM. As a result, although buyers cannot directly detect δ , they could infer that only sellers with $\delta = 0$ would show up in DM. Without loss of generality, we assume buyers always propose the contract as $\{q(x, \delta) = 1, \tau(x, \delta) = \eta x\}$ after paying information cost κ . Then the objective function of seller- (x, δ) is formulated as below.

$$U^S(x, \delta) = \max_{\{CM, DM, Delay\}} \{c_1 + \delta \cdot c_2\}$$

where

$$\begin{aligned}
a &= \begin{cases} 1 & \text{sell the asset at } t = 1 \\ 0 & \text{keep it until } t = 2 \end{cases} \\
c_1 &= \max_{\{DM, CM\}} \left\{ \frac{m(b, s)}{s} \cdot \eta x, p(x) \right\} \cdot a \\
c_2 &= a \cdot \mathbf{1}_{\{\frac{m(b, s)}{s} \cdot \eta x > p(x)\}} \cdot \left[1 - \frac{m(b, s)}{s} \right] \cdot x + (1 - a) \cdot x
\end{aligned}$$

and b and s denote the measure of buyers and that of sellers in DM respectively, and $p(x)$ the price of asset- x in CM. Notice that we employ random search in the benchmark. Thus (b, s) does not differentiate the measure of trading parties in certain sub-markets. For sellers- $(x, \delta = 0)$, they have to sell their asset at $t = 1$. Thus their discrete choice is reduced to $\max_{\{DM, CM\}} \left\{ \frac{m(b, s)}{s} \eta x, p(x) \right\}$. For sellers- $(x, \delta = 1)$, in addition to participating in either DM or CM at either $t = 1$, they could also exercise the option of waiting until the dividend is delivered at $t = 2$. Thus their target is formulated as $\max_{\{DM, CM, Delay\}} \left\{ \frac{m(b, s)}{s} \eta x + \left[1 - \frac{m(b, s)}{s} \right] x, p(x), x \right\}$.

2.2.3 Choice of Trading Venues

This part analyzes the choice of trading venues under both complete and incomplete information. For complete information, we mean both asset payoffs x and liquidity shock δ , *i.e.*, trading motives, are publicly observable.

Complete Information

Denote $p(x)$ as the price of asset- x in CM. Since buyers are fully competitive, buyer's profit $x - p(x)$ from buying asset- x should be zero. As a result, $p(x) = x$ for all x with complete information. Then we reach the following proposition.

Proposition 4. (Market Participation under Complete Information) When there is no information asymmetry,

1. Any seller- $(x, \delta = 0)$ would prefer CM to DM for asset trading at $t = 1$.
2. Any seller- $(x, \delta = 1)$ would never try DM. They are indifferent between trading in CM at $t = 1$ and waiting to consume at $t = 2$.

3. None of buyers incur information investment. Instead, all of them concentrate in CM in $t = 1$.

The key message of this proposition is, when there is no information asymmetry on asset payoffs, the CM is preferred to search frictions and bargaining in DM. We move on to the discussion with information asymmetry in the rest of this section.

Incomplete Information

When (x, δ) are private information of sellers, $p(x)$ is the same for sellers self-selecting to pool in CM at $t = 1$. Denote $p(x) = p$ in this case. Then we have the following result on choice of trading venues in partial equilibrium.

Proposition 5. (Market Participation under Two-dimensional Information Asymmetry) When *both* asset payoff and liquidity shock are seller's private information, market participation is a choice function of seller- (x, δ) from $X \times \Delta = [x_L, x_H] \times \{0, 1\}$ to $\{CM, DM, Autarky\}$ such that,

1. For sellers with $\delta = 0$, there exists a cut-off point $\tilde{x} \in [x_L, x_H]$ such that if $x \geq \tilde{x}$, they would self-select into DM, and enter CM otherwise at $t = 1$.
2. For sellers with $\delta = 1$, if $x < p$, they would choose CM, and if $x \geq p$, they would participate in neither DM nor CM at $t = 1$, but instead wait to consume at $t = 2$.
3. Given (\tilde{x}, p) , the utility function of seller- (x, δ) is refined as below.

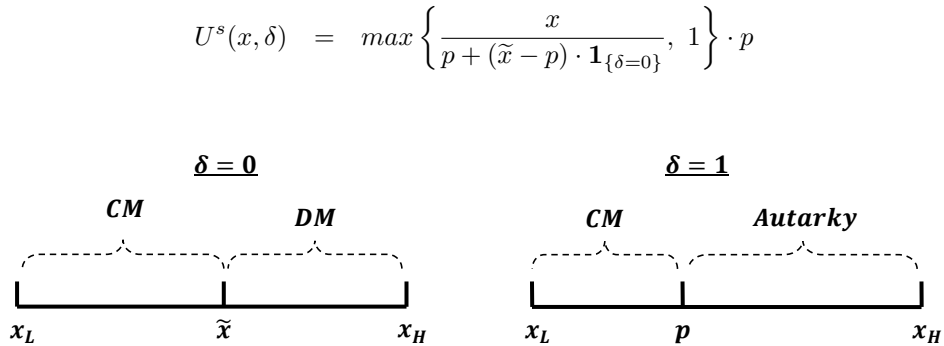


Figure 2.2: Choice of trading venues by seller- (x, δ)

The main message of this proposition is, when sellers are subject to preference shock, those with high-quality assets tend to sell at DM while the others pool into CM. However, for those without preference shock, since they have the outside option as waiting and consuming by themselves, they would never try DM with search and bargaining. Meanwhile, they would take advantage of CM if their asset's quality is low. For illustration concern, I summarize the choice of trading venues by seller- (x, δ) and associated gain $U^s(x, \delta)$ in Figures 2.2 and 2.3 respectively.

Figure 2.2 immediately implies the measure of sellers in DM is

$$s = \pi \cdot [1 - F(\tilde{x})], \quad (2.1)$$

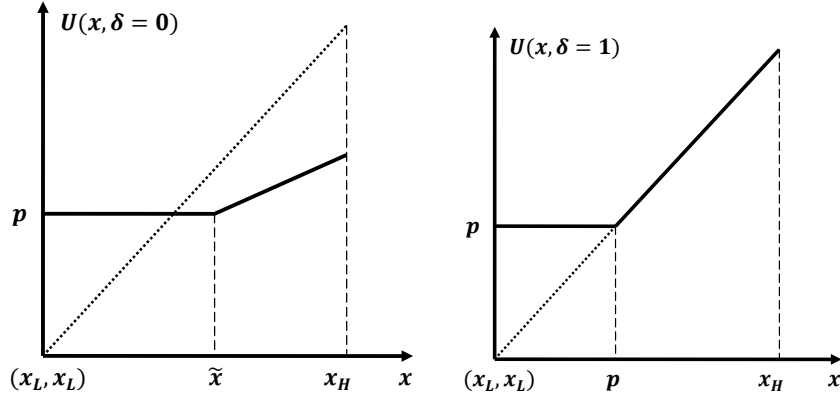


Figure 2.3: Expected gain of seller- (x, δ)

Finally, based on Proposition 2, we show the results on market participation when either x or δ is private information.

Corollary 10. (Market participation with only one-dimensional information asymmetry) When either asset payoff and liquidity shock is seller's private information, market participation of sellers is as below.

1. **(When only trading motive is private information)** In this case, the result is the same as that with complete information on (x, δ) in Proposition 1. That is,

- (a) Any seller- $(x, \delta = 0)$ would always prefer CM to DM for asset trading at $t = 1$.
- (b) Any seller- $(x, \delta = 1)$ would never try DM. They are indifferent between trading in CM at $t = 1$ and waiting to consume at $t = 2$.
- (c) None of buyers incur information investment. Instead, all of them concentrate in CM in $t = 1$.

2. **(When only asset payoff is private information)** In this case, the result is very similar to the that of Proposition 2, but with $\pi = 1$.

- (a) For sellers with $\delta = 0$, there exists a cut-off point $\tilde{x}^* \in [x_L, x_H]$ such that if $x \geq \tilde{x}^*$, they would self-select into DM, and enter CM otherwise at $t = 1$.
- (b) For sellers with $\delta = 1$, if $x < p$, they would choose CM, and if $x \geq p$, they would participate in neither DM or CM at $t = 1$, but instead wait to consume at $t = 2$.
- (c) Buyers who are posting price p in CM would never accept sellers with $\delta = 1$. Thus the equilibrium result would perform as if all sellers transferring their assets are $\delta = 0$, *i.e.*, $\pi \equiv \Pr(\delta = 0) = 1$.

That is, if only δ or x serves as private information, seller's choice of trading venues is reduced to the case with complete information and that with two-dimensional informational asymmetry in Proposition 1 and 2 respectively. As a result, this corollary justifies why we stick to the general case with both x and δ as being seller's private information when departing from the case with complete information. Moreover, we would rely on this corollary to simplify our argument in Section 4 on welfare analysis by assuming only asset payoff is private information.

2.2.4 Asset Price in Centralized Market

There are two key variables in our partial-equilibrium analysis. One is p , the price in CM while the other one is \tilde{x} , the cut-off point of choice between CM and DM. We

use this section and the next one to reach two equations to determine (p, \tilde{x}) . Since buyers are assumed to be competitive in CM, none of them make positive profits in equilibrium. Thus the price in CM is determined as below.

$$p = \frac{\pi F(\tilde{x}) \mathbb{E}(x|x \leq \tilde{x}) + (1 - \pi) F(p) \mathbb{E}(x|x \leq p)}{\pi F(\tilde{x}) + (1 - \pi) F(p)} \quad (2.2)$$

The LHS of Eq. (2.2) is buyer's cost for one unit of asset. The RHS is the average value of assets pooling in CM. Buyers are uninformed of the true value of each asset in CM. However, as implied in the above proposition, buyers have rational expectation of $F^{CM}(x)$, the true (truncated) distribution of asset payoffs in CM. One source is from those sellers with $(x \leq \tilde{x}, \delta = 0)$ while the other one from those with $(x \leq p, \delta = 1)$. The numerator and the denominator of Eq. (2.2) are the total value and total measure of assets in CM respectively.

Lemma 4. (Asset Price in CM) Given any (\tilde{x}, π) , Eq. (2.1) has a unique solution as $p = P_{AS}(\tilde{x}, \pi)$.

1. For the general case, we have

- (a) $\partial P_{AS} / \partial \tilde{x} > 0$ and $\partial P_{AS} / \partial \pi > 0$.
- (b) $x_L = P_{AS}(\tilde{x} = x_L, \pi) \leq p \leq P_{AS}(\tilde{x}, \pi = 1) = \mathbb{E}(x|x \leq \tilde{x}) \leq \min\{\tilde{x}, \mu\}$.
- (c) $P_{AS}(\tilde{x}, \pi = 0) = x_L$. Thus CM completely collapses when $\pi = 0$.

2. When $x \stackrel{U}{\sim} X = [x_L, x_H]$, we have

$$p = P_{AS}(\tilde{x}, \pi) = \varphi(\pi) \cdot \tilde{x} + [1 - \varphi(\pi)] \cdot x_L, \quad (2.3)$$

where $\varphi(\pi) \equiv \frac{\sqrt{\pi}}{\sqrt{\pi}+1}$ and $Pr\{\delta = 0\} = \pi \in [0, 1]$.

Several comments are made here. First of all, when $\pi = 1$, *i.e.*, all sellers would be hit by liquidity shock at $t = 1$, then $p = P_{AS}(\tilde{x}, \pi = 1) = \mathbb{E}(x|x \leq \tilde{x})$, a classic problem on adverse selection. Secondly, when $\pi = 0$, *i.e.*, all sellers pooling in CM is simply due to selling lemons rather than liquidating for liquidity need, then CM simply collapses because of severe adverse selection.

2.2.5 Free Entry of Information Investment

To make the analysis on market co-existence non-trivial, we have to assume buyer's information investment κ is small relative to the average asset quality. Otherwise, buyers would have no incentive to pay the cost and trade in the DM. In anticipating this scenario, both sellers and buyers would always concentrate in CM. Buyer's free entry condition in DM is then formulated as below.

$$\begin{aligned} \left[\frac{m(b, s)}{b} (1 - \eta) \mathbb{E}(x|x \geq \tilde{x}) - \kappa \right] \cdot b &= 0, \\ \frac{m(b, s)}{b} (1 - \eta) \mathbb{E}(x|x \geq \tilde{x}) - \kappa &\leq 0, \quad b \geq 0, \end{aligned}$$

where $s = \pi[1 - F(\tilde{x})]$. Assume DM exists, *i.e.*, $b > 0$ and $s > 0$, and denote the market tightness as $\alpha \equiv \frac{s}{b}$, then we have

$$m(1, \alpha)(1 - \eta)r(\tilde{x}) = \kappa. \quad (2.4)$$

Since $r(\tilde{x}) \equiv \mathbb{E}(x|x \geq \tilde{x})$ increases with \tilde{x} and $m(1, \alpha)$ increases with α , applying Implicit Function Theorem to the above equation immediately suggests a negative relationship between α and \tilde{x} . I denote it as $\alpha = \alpha(\tilde{x})$, a decreasing function of \tilde{x} . In turn, when $\tilde{x} \in (x_L, x_H)$, by definition it is characterized by $\frac{m(b, s)}{s} \eta \tilde{x} = p$, which can be further refined as below.

$$p = P_{FE}(\tilde{x}) \equiv \underbrace{m\left(\frac{1}{\alpha(\tilde{x})}, 1\right)}_{ext. \text{ margin}} \cdot \underbrace{\eta \tilde{x}}_{int.} \quad (2.5)$$

Since $\alpha(\tilde{x})$ decreases with \tilde{x} , the LHS of Eq. (2.3) delivers a positive relationship between \tilde{x} and p . We denote it as $p = P_{FE}(\tilde{x})$. It is worth noting that, since $m(1, \alpha) < \lambda$, to guarantee that $\alpha(\tilde{x})$ always exists, we must have $\mathbb{E}(x|x \geq \tilde{x}) > \frac{\kappa}{\lambda(1-\eta)}$. A sufficient condition is $\mathbb{E}(x|x \geq x_L) = \mu > \frac{\kappa}{\lambda(1-\eta)}$. If $\mathbb{E}(x|x \geq \tilde{x}) \leq \frac{\kappa}{\lambda(1-\eta)}$ holds for some \tilde{x} , then we have a corner solution as $b = \alpha(\tilde{x}) = 0$. An extreme case is that, if $x_H = \mathbb{E}(x|x \geq x_H) \leq \frac{\kappa}{\lambda(1-\eta)}$, then $\alpha(\tilde{x}) = 0$ for all $x \in X \equiv [x_L, x_H]$. Another comment is, when $\tilde{x} = x_H$, we have $s = 0$ and thus we always have $b = 0$ in this case.

Lemma 5. (Buyer's Free Entry Condition on Information Investment)

1. Given any p , there exists a unique \tilde{x} which satisfies Eq. (2.3) and is denoted as $\tilde{x} = X_{FE}(p; \lambda, \kappa, \eta)$. X_{FE} and P_{FE} have the following property.

$$\partial X_{FE}/\partial p > 0, \partial X_{FE}/\partial \kappa > 0, \partial X_{FE}/\partial \eta < 0, P_{FE}(x_L) < x_L.$$

2. When $m(b, s) = \lambda \cdot \min\{b, s\}$ and $x \overset{U}{\sim} X = [x_L, x_H]$,

- (a) Eq. (2.3) is simplified as

$$\lambda \cdot \min\{1, \alpha\}(1 - \eta)r(\tilde{x}) = \kappa, \quad (2.6)$$

where $\alpha \equiv \frac{s}{b}$ and $r(\tilde{x}) \equiv \mathbb{E}(x|x \geq \tilde{x}) = \frac{\tilde{x} + x_H}{2}$.

- (b) The marginal seller between CM and DM is characterized as

$$\lambda \cdot \min\left\{\frac{1}{\alpha}, 1\right\} \cdot \eta \cdot \tilde{x} = p. \quad (2.7)$$

When p increases, terms of trade in CM becomes more favorable for sellers with $\delta = 1$ and thus more of them switch to CM. In the same spirit, when κ increases, given any \tilde{x} , the market tightness would be more tough for sellers and thus some of them would then pool into CM. Moreover, when η increases, the terms of trade in DM looks more attractive and thus sellers in CM would then switch to DM.

In this subsection, we have so far focused on the scenario of market co-existence. That is, we implicitly assume that $\tilde{x} \in (x_L, x_H)$ and $s, b > 0$. When $\tilde{x} = x_H$, *i.e.*, only CM exists, then $s = 0$ by Eq. (2.1). In turn, the free entry condition implies that $b = 0$. In this case, we have

$$\tilde{x} = X_{FE}(p; \lambda, \kappa, \eta) = x_H, \quad (2.8)$$

which holds for all $p \geq 0$. However, another polar case with $\tilde{x} = x_L$ can never be possible. Here is the reasoning. Since the lower bound of asset payoff is $x_L > 0$ and the competitive buyers would offer $p \geq x_L$, for sellers with $x = x_L$, they would always prefer the immediacy of CM to the search and bargaining in CM. By continuity, for sellers with x being close to $x = x_L$ would always choose CM over DM. Therefore it could never be that CM is totally replaced by DM unless $x_L = 0$.

2.3 Equilibrium Choice of Trading Venues

2.3.1 General Equilibrium

Combining *AS* and *FE* condition solves (p, \tilde{x}) , which is illustrated in Figure (2.4).

Then we reach the general equilibrium choice of trading venues as below.

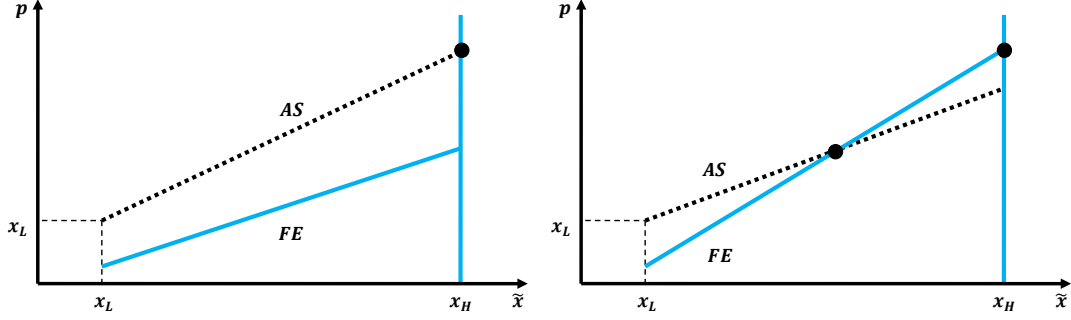


Figure 2.4: (p, \tilde{x}) are jointly determined by the intersection AS-curve and FE-curve.

Definition. The equilibrium consists of (i) p , price in CM, \tilde{x} , the cut-off point of the choice of trading venues, b , the measure of buyers making information investment and entering DM, s , the measure of sellers self-selecting into DM, α , the market tightness of DM at $t = 1$; (ii) seller's choice of trading venues at $t = 1$, such that

1. Given (p, \tilde{x}) , seller's choice of trading venues is characterized by Figure 2.2.
2. (p, \tilde{x}) are jointly determined by Eq. 2.3, Eq. (2.3) and Eq. (2.8).
3. Given p and \tilde{x} , s is given by (2.1) α by Eq. (2.4) and b in turn by $b = s/\alpha$.¹⁴

We use the following proposition to fully characterize the equilibrium choice of trading venues.

Proposition 6. (Equilibrium Choice of Trading Venues) Assume $x \overset{U}{\sim} X = [x_L, x_H]$ and $m(b, s) = \lambda \cdot \min\{b, s\}$. Denote $\underline{\kappa} \equiv \frac{\lambda(1-\eta)}{2} \left[\left(\frac{1-\varphi}{\lambda\eta-\varphi} \right) x_L + x_H \right]$ and $\bar{\kappa} \equiv \lambda(1-\eta)x_H$.

¹⁴If $\tilde{x} = x_H$ in equilibrium, we have $b = s = 0$ and then the market tightness $\alpha = s/b$ is not well-defined, but it would not bother our analysis then.

1. When $\lambda\eta > \varphi$ and $\sigma > \hat{\sigma} \equiv (\frac{1-\lambda\eta}{1+\lambda\eta-2\varphi})\mu$ (i.e., $\lambda\eta > \varphi + (1-\varphi)\frac{x_L}{x_H}$), where $\varphi = \varphi(\pi) \equiv \frac{\sqrt{\pi}}{\sqrt{\pi}+1}$, we have $\bar{\kappa} > \underline{\kappa}$ and

$$\tilde{x} = \min \left\{ x_H, \max \left\{ \left(\frac{1-\varphi}{\lambda\eta-\varphi} \right) x_L, \frac{2\kappa}{\lambda(1-\eta)} - x_H \right\} \right\} = \begin{cases} x_H & \text{if } \kappa > \bar{\kappa} \\ \frac{2\kappa}{\lambda(1-\eta)} - x_H & \text{if } \underline{\kappa} < \kappa \leq \bar{\kappa} \\ \left(\frac{1-\varphi}{\lambda\eta-\varphi} \right) x_L & \text{if } \kappa \leq \underline{\kappa} \end{cases}$$

$\tilde{x} = x_H$ can be also supported in this case, but it is not stable.

2. When $\lambda\eta \leq \varphi$ or $\sigma \leq \hat{\sigma}$ (i.e., $\lambda\eta \leq \varphi + (1-\varphi)\frac{x_L}{x_H}$), we have $\tilde{x} = x_H$ for all $\kappa \in \mathbb{R}_+$.

We use Figure 2.5 to illustrate the above proposition. Intuitively, when κ is large enough, i.e., $\kappa > \bar{\kappa}$, even though matching efficiency is high in DM and adverse selection is low in CM, only CM would survive for asset trading. When κ decreases, it is not only more likely that market co-existence could emerge, but also more trading would switch to DM given the co-existence is sustained. The upper panel of Figure 2.5 treat other exogenous variables as given and focus on the effect of information cost κ on equilibrium choice of trading venues.

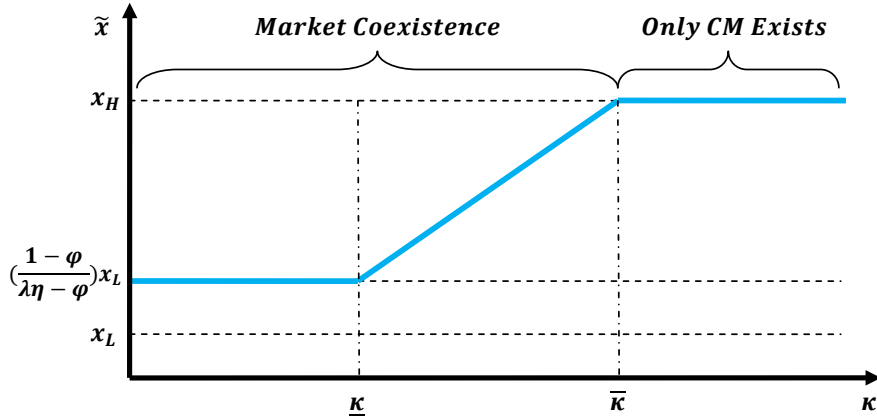


Figure 2.5: **Equilibrium Choice of Trading Venues** ($\lambda\eta > \varphi$ and $\sigma > \hat{\sigma} \equiv (\frac{1-\lambda\eta}{1+\lambda\eta-2\varphi})\mu$).

Based on Figure 2.5, given any κ , we use the upper panel to of Figure (2.6) demonstrate the implication of the increase of matching efficiency in DM for choice

of trading venues. It is clearly shown that, holding κ constant, both extensive-margin and intensive-margin changes with λ . The second panel suggests that \tilde{x} increases with π . The intuition is, when π decreases, the adverse selection tends to be more severe in CM and thus DM looks more attractive for high-quality sellers and thus \tilde{x} decreases. The first three panels illustrates the monotone relationship between \tilde{x} and (κ, λ, π) respectively. In contrast, the lower panel suggests the relationship between (η, \tilde{x}) is not monotone. When seller's bargaining power η increase, which may be due to the increasing competition of buyers in DM, the direct effect is that \tilde{x} would decrease since the terms of trade in DM looks more attractive. Meanwhile, when η increases, the proportion of what buyers could get from trading in DM would decrease and thus they have less incentive to enter. In turn, seller's matched probability in DM would decrease, which would discourage sellers from choosing DM over CM. That is, \tilde{x} would increase in response to the second effect. The lower panel implies that the first effect is dominant when information cost κ is low enough while just opposite when κ is high.

2.3.2 Trading Share and Distribution of Asset Payoff

After obtaining the equilibrium values on (p, \tilde{x}) , we obtain the determinants of trading share in both markets. Moreover, we characterize the distribution of asset payoffs in CM and DM.

Trading Share in CM and DM

According to Figure 2.2, the measure of sellers participating in either CM or DM at $t = 1$ is $\omega = \pi + (1 - \pi)F(p)$. As a result, *conditioning on* trade exercised at $t = 1$ and using Eq. (2.9), the (truncated) trading share in CM is

$$\rho^{CM} = \frac{\pi F(\tilde{x}) + (1 - \pi)F(p)}{\pi + (1 - \pi)F(p)} = \frac{\pi \tilde{x} + (1 - \pi)p - x_L}{\pi x_H + (1 - \pi)p - x_L}, \quad (2.9)$$

First of all, if $\lambda\eta \leq \varphi$ or $\sigma \leq \hat{\sigma}$, as implied by Proposition 5, we always have $\rho^{CM} = 1$. Secondly, if $\lambda\eta > \varphi(\pi)$, we have

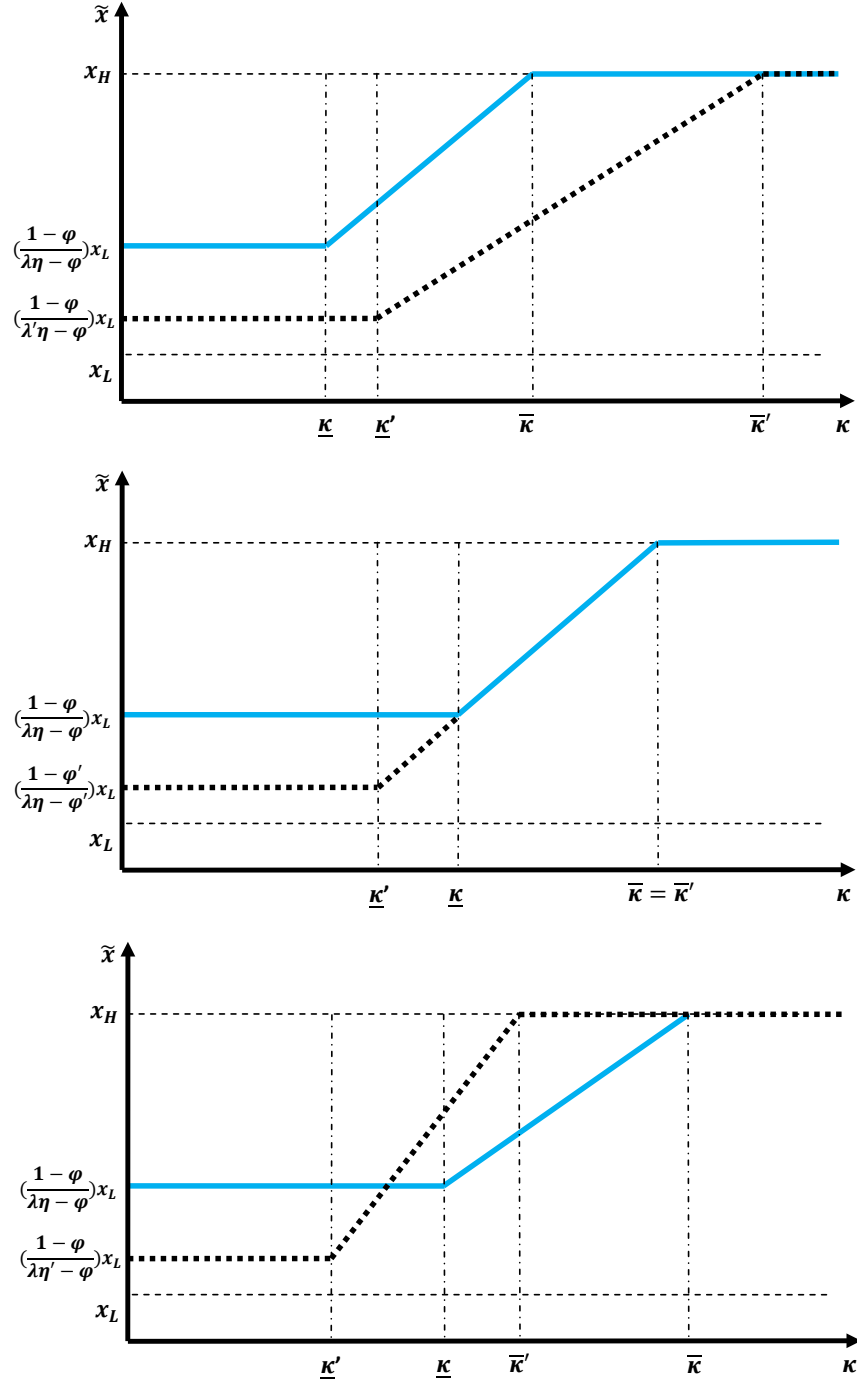


Figure 2.6: **Top:** when λ increases; **middle:** when π decreases; **bottom:** when η increases.

$$\rho^{CM} = \begin{cases} 1 & \text{if } \kappa > \bar{\kappa} \\ \frac{\frac{2[\pi+\varphi(1-\pi)]}{\lambda(1-\eta)}\kappa - 2\varphi(1-\pi)\mu - 2\pi\mu}{\frac{2\varphi(1-\pi)}{\lambda(1-\eta)}\kappa - 2\varphi(1-\pi)\mu} & \text{if } \underline{\kappa} < \kappa \leq \bar{\kappa}, \\ \frac{[\pi+(1-\pi)\lambda\eta](\frac{1-\varphi}{\lambda\eta-\varphi})-1}{\pi(\frac{\mu+\sigma}{\mu-\sigma})+(1-\pi)\lambda\eta(\frac{1-\varphi}{\lambda\eta-\varphi})-1} & \text{if } \kappa \leq \underline{\kappa} \end{cases} \quad (2.10)$$

Therefore, in the presence of market co-existence, we have the following comparative statics.

$$\frac{\partial \rho^{CM}}{\partial \mu} \geq 0 \quad \frac{\partial \rho^{CM}}{\partial \sigma} \leq 0 \quad \frac{\partial \rho^{CM}}{\partial \kappa} \geq 0 \quad \frac{\partial \rho^{CM}}{\partial \lambda} \leq 0 \quad \frac{\partial \rho^{CM}}{\partial \eta} ? 0 \quad \frac{\partial \rho^{CM}}{\partial \pi} \geq 0$$

As implied in Eq. (2.9), the effect of λ , η , etc. on ρ^{CM} is through their impact on \tilde{x} , which in turn works on ρ^{CM} . First of all, when the adverse selection is alleviated, *i.e.*, $\frac{\sigma}{\mu}$ decreases, the trading share in CM increases. Secondly, when λ increases, say, due to IT improvement, the DM tends to be more attractive for sellers and thus the trading share in CM shrinks. The same logic applies to the argument on the effect of π on ρ^{CM} . Thirdly, when π increases, the proportion of sellers with preference shock rather than selling lemons increases. The average quality of assets in CM increases and thus more sellers would trade in CM, which boosts ρ^{CM} . Finally and again, since the information cost κ has no role in neither \tilde{x} nor p due to the specification on matching function, it does not affect ρ^{CM} provided κ is low enough. In general, when κ decreases, due to financial deregulation or IT improvement, DM tends to absorb more sellers, *i.e.*, \tilde{x} would decrease and low ρ^{CM} . In sum, the exercise of comparative statics ρ^{CM} lends us insight on the secular migration of bond trading in the past century, which is well documented by Biais and Green (2007). However, it is worth noting that the sign of $\frac{\partial \rho^{CM}}{\partial \eta}$ is ambiguous. Here is the intuition. On one hand, when η increases, the terms of trade in the intensive margin looks more attractive to sellers. On the other hand, the increase of η discourages buyers from making information investment in the extensive margin. In turn, it would be less likely for sellers to be matched with buyers in DM. It is the trade-off between intensive and extensive margin by η that makes ρ^{CM} not monotone with η .

Distribution of Asset Payoffs

First, given market co-existence, the truncated distributions of asset quality in DM and that in CM are given respectively as below.

$$F^{DM}(x) = \begin{cases} 1 & \text{when } x \in (x_H, +\infty) \\ \frac{F(x)-F(\tilde{x})}{1-F(\tilde{x})} & \text{when } x \in (\tilde{x}, x_H] \\ 0 & \text{when } x \in (-\infty, \tilde{x}] \end{cases} \quad F^{CM}(x) = \begin{cases} 1 & \text{when } x \in (\tilde{x}, +\infty) \\ \frac{\pi F(x)+(1-\pi)F(p)}{\pi F(\tilde{x})+(1-\pi)F(p)} & \text{when } x \in (p, \tilde{x}] \\ \frac{F(x)}{\pi F(\tilde{x})+(1-\pi)F(p)} & \text{when } x \in (x_L, p] \\ 0 & \text{when } x \in (-\infty, x_L] \end{cases}, \quad (2.11)$$

and thus $F^{DM}(x) \leq F(x) \leq F^{CM}(x)$. That is, in the presence of adverse selection, high-quality assets tend to be sold in DM while low-quality ones prefer the immediacy of CM. Bolton, Santos and Scheinkman (2011b) document a similar theoretical finding. However, it is worth mentioning that, as argued in the literature review, the information structure differs in our papers and the co-existence of CM and DM is endogenous.

Secondly, when only CM exists for asset trading, $\tilde{x} = x_H$ and thus $F^{DM}(x)$ is degenerate. $F^{CM}(x)$ is modified as below.

$$F^{CM}(x) = \begin{cases} 1 & \text{when } x \in (x_H, +\infty) \\ \frac{\pi F(x)+(1-\pi)F(p)}{\pi+(1-\pi)F(p)} & \text{when } x \in (p, x_H] \\ \frac{F(x)}{\pi+(1-\pi)F(p)} & \text{when } x \in (x_L, p] \\ 0 & \text{when } x \in (-\infty, x_L] \end{cases}. \quad (2.12)$$

2.3.3 Aggregation with Information Investment

We close this section with a remark on information use. Since our model only considers an exchange economy, the aggregate asset payoffs are fixed. The aggregation with every sellers having the same weight simply suggests that information investment by buyers is a waste of social resources. Following this line of argument, forbidding trade in DM is seemingly socially desirable. Moreover, we can reply on Proposition 3 to obtain the equilibrium values on p , the price in CM, as well as q , the weighted revenue. We illustrate both of them in Figure (2.7). That is, the emergence of DM with costly information acquisition dampens both the liquidity in

CM and the average asset revenues.

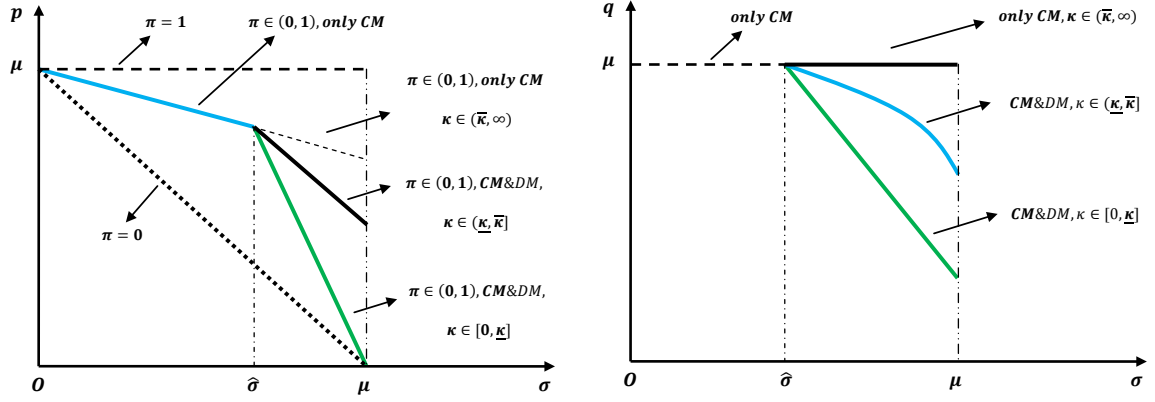


Figure 2.7: **Price in CM (p) and the Weighted Asset Price (q)**

However, since sellers are ex ante heterogeneous in asset payoffs and liquidity shock, the aforementioned simple weighted calculation is misleading to some extent. As shown in Proposition 3, market co-existence can be sustained under some conditions. In this scenario, sellers with high-quality asset, prefer to bear search friction in DM rather than subsidize low-quality assets in CM. As a result, closing DM would make those sellers worse off. The discussion on the government asset purchase program equips us with a further illustration of this observation.

2.4 Government Asset Purchase Program

Prior to the financial recession, MBS was considered to be information insensitive assets and thus there did not appear to exist an information asymmetry. However, the outbreak of the financial crisis reminded the market of the potential information asymmetry within the MBS market. Consequently, financial markets tended to be illiquid and some markets, such as the federal funds market, were also frozen. See Heider, Hoerova and Holthausen (2010) and Gale and Yorulmazer (2013) among others for the background description and theoretical explanation.

The US government launched the Troubled Asset Relief Program (TARP) to curb the recent financial crisis. More specifically, the US Treasury implemented the TARP by purchasing mortgage backed securities (MBS) from the financial institutions.¹⁵ In this section, we use the baseline model to address the implication of government intervention for the seller's welfare. We focus on the seller's welfare since buyers are assumed to be fully competitive and thus they would make zero profit in equilibrium. In particular, we raise the following question. Does a self-financing government intervention make all sellers better off? If not, how would the heterogeneous treatment effect be related to the seller's asset quality?

Thanks to Corollary 1, we can concentrate on the simplified case with $\pi = 1$, *i.e.*, all sellers are hit by liquidity mismatch and thus have to sell their assets to buyers before the asset payoffs are realized. Then we can index each seller as seller- x rather than seller- (x, δ) in the baseline. Due to the free entry condition of information investment and trading in the decentralized market (DM), if the government has to incur a higher information cost than do the normal buyers in the baseline, or if its matching efficiency in the DM is lower, then the government would make a loss from its intervention. To make the analysis non-trivial, we assume the asset purchase program is self-financing. In turn we make the following assumption.¹⁶

Assumption 3. The government enjoys a lower information cost than buyers, and market co-existence is always sustainable, *i.e.*, $\kappa_g < \kappa_b < \bar{\kappa} \equiv \lambda(1 - \eta)x_H$.

To implement the program, the government issues perfectly enforceable debts to buyers at the beginning of $t = 1$. Thus the government receives consumption goods produced by buyers. When government steps into asset markets, it does not necessarily have an information advantage over the uninformed buyers in the baseline on asset payoffs. We adopt a more reasonable assumption by treating the government in a similar position as uninformed buyers. That is, the government could always set up a pooling price in the CM. Alternatively, the government can

¹⁵See the following link for more details of this program: <http://www.federalreserve.gov/bankinfo/tarinfo.htm>.

¹⁶Alternatively, we could assume $\lambda_g > \lambda_b$, *i.e.*, the government would enjoy a higher matching efficiency in DM after paying the same information cost. Moreover, we can easily relax the assumption that $\kappa_b < \bar{\kappa}$.

make an information investment and kick off bilateral trade with sellers in the DM. They can also launch the trade in both markets in the same time.

In sum, with these consumption goods at hand, the government buys seller's assets in the CM, and decides whether or not to pay the information cost and buy assets from DM. At $t = 2$, the government receives consumption goods from the pooling assets it purchases from CM (and DM, if it co-exists with CM) at $t = 1$. The government clears its liabilities by repaying buyers with the goods. Since buyers are fully competitive, buyers make zero profit just like the self-financing government intervention does.

On the one hand, since the information cost of government is lower than that of the normal buyers, the free entry condition on information investment in DM suggests that only the government survives in asset exchange in DM with information investment. On the other hand, since the government is self-financing and buyers are fully competitive, neither of them gain positive profit from trading in CM. Without loss of generality, we assume only the government trades with sellers in the CM. Therefore in the presence of Assumption 1, only the government would trade with sellers in either markets in equilibrium. We summarize the key findings in the following proposition.

Proposition 7. (Welfare Effect of Government Asset Purchase Program)

Under Assumption 1, a self-financing government asset purchase program makes high-quality sellers better off while the low quality sellers worse off. More specifically, there exists a cutoff point $\hat{x} \in (x_L, x_H)$ such that,

1. Sellers with $x \geq \hat{x}$ are better off. Moreover, the net gain strictly increases with their asset quality x .
2. Sellers with $x < \hat{x}$ are worse off. Moreover, the net loss weakly increases with their asset quality x .

The above proposition states that, with Assumption 1, *i.e.*, even though the government has an information advantage than the normal buyers, the government cannot deliver a Pareto improvement for the heterogeneous sellers. The decrease of

the information cost by the government encourages it to acquire more information in the DM. As a result, sellers who stay in the DM after the government intervention enjoy a more favorable extensive margin. Moreover, the favorable market tightness in general equilibrium drives more sellers to switch from the CM to the DM. Therefore the average quality of assets in the CM decreases. In turn, the pooling price in the CM decreases and those who continue to trade in the CM are worse off. Consequently some sellers are better off while the others are worse off. We illustrate the logic and the cut-off value of the above proposition in Figure 2.8.

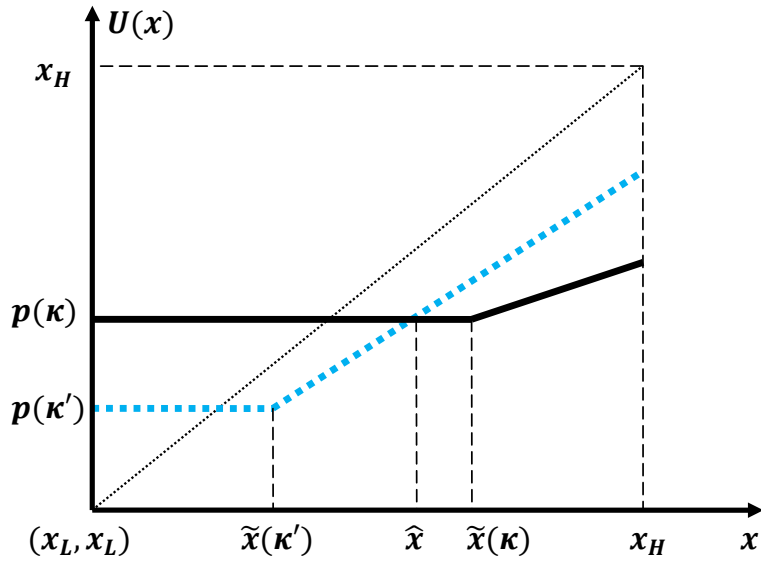


Figure 2.8: When government steps in with a lower information cost ($\kappa' < \kappa$).

2.5 Conclusion

Asset exchange with market co-existence is prevalent. To this end, we develop a simple model to characterize conditions under which co-existence of centralized and decentralized markets can be sustained and its implication for asset liquidity. There are two-dimensional private information, one is asset payoff while the other one is liquidity shock of asset sellers. On one hand, the latter dimension is always unobservable by others. Buyers can either stay uninformed or choose to acquire costly

information on the former dimension. If buyers incur no information cost, then they post a pooling and publicly displayed price at which asset demand equals supply. This is what we mean by centralized market (CM), which is free of search friction, but is subject to adverse selection. In contrast, those buyers with costly information acquisition may propose a trading menu different from the pooling price. Since we assume each information investment can only be used to detect the quality of one unit of asset, the bilateral trading between sellers and informed buyers may be subject to search frictions. This is what we mean by decentralized market (DM), which is characterized with search friction and bilateral bargaining. That is, the endogenous information investment delivers the emergence of DM with bilateral trading.

Due to strategic complementarity between sellers and buyers, there always exists an equilibrium in which only CM survives for asset exchange. To ease the analysis comparative statics, we always pick up the equilibrium with markets co-existence whenever it can be supported. Market co-existence emerges only when the following three conditions are satisfied: i) the search friction in DM is low enough, ii) the information friction in CM is severe enough, and iii) the information cost is low enough. Given market co-existence, the trading share of DM over CM increases with matching efficiency in DM and severeness of adverse selection in CM, while decreases with information cost. Then we conclude that, as matching efficiency in DM increases and the information cost decreases, more trade migrates from CM with adverse selection to DM with search frictions. In the limit, DM with search frictions converges to CM with complete information.

Our model with information and search frictions is more than just explaining conditions under which CM and DM co-exist for asset trading. We also address the implication of government asset purchase program, such as the Troubled Asset Relief Program (TARP), through the lens of our model. Since sellers are heterogeneous in asset payoffs, even though the government is better at information cost or matching efficiency, the treatment effect of self-financing government asset purchase program is heterogeneous. Moreover, in the presence of government intervention, we show that sellers with high-quality assets are better off while the others are worse off in general equilibrium. Therefore even though the government has an information advantage

than the normal buyers, the government cannot generate a Pareto improvement for all those heterogeneous sellers.

We close the paper with several possible lines for future research. First, we assume assets are indivisible. This assumption is innocuous in the paper since we also assume all agents are risk neutral. Both restrictions contribute greatly to tractability. It could be interesting to extend the idea into the scenario with perfect divisible asset. The advancement from indivisibility to divisibility is not a trivial exercise. As emphasized by Lagos and Rocheteau (2009), “...As a result of the restrictions they imposed on asset holdings, existing search-based theories neglect a critical feature of illiquid markets, namely, that market participants can mitigate trading frictions by adjusting their asset positions to reduce their trading needs...”. Both Lagos and Wright (2005) and Lagos and Rocheteau (2009) have contributed a tractable framework for asset trading with perfectly divisible asset.

Secondly, to neatly model endogenous information acquisition and the emergence of DM, we assume direct trading between sellers and buyers in a finite-horizon model. In our real life, however, a large number of asset trading in DM are dealer-intermediated, say corporate bonds. To better characterize the trading details in DM, such as bid-ask spread, it may be worthwhile for us to introduce dealer between sellers and buyers in DM.

Thirdly, it might be desirable for us to integrate the idea in this paper into a dynamic general equilibrium model. Eisfeldt (2004) and Kurlat (2012), among others, are excellent examples of integrating pooling price with adverse selection into RBC models. As suggested throughout this paper, buyers in our paper undertake endogenous level of information investment to lessen adverse selection. Furthermore, we have endogenous trading venues for market liquidity. In sum, the RBC model with our story might deliver additional insights for dynamic decision on real investment and information investment and their interactions with each other.

2.6 Appendix

2.6.1 Appendix A - Proofs

Proof of Proposition 1 and 2

Proof. Substituting c_1 and c_2 into the objective function yields

$$\begin{aligned}
 U^S(x, \delta) &= \max_{a \in \{0,1\}} \left\{ \max \left\{ \frac{m(b,s)}{s} \eta x, p(x) \right\} \cdot a + \delta \cdot \left[a \cdot \mathbf{1}_{\left\{ \frac{m(b,s)}{s} \cdot \eta x > p(x) \right\}} \cdot \left(1 - \frac{m(b,s)}{s} \right) \cdot x + (1-a) \cdot x \right] \right\} \\
 &= \begin{cases} \max_{a \in \{0,1\}} \left\{ \max \left\{ \frac{m(b,s)}{s} \eta x, p(x) \right\} \cdot a \right\} & \text{when } \delta = 0 \\ \max_{a \in \{0,1\}} \left\{ \max \left\{ \frac{m(b,s)}{s} \eta x, p(x) \right\} \cdot a + a \cdot \mathbf{1}_{\left\{ \frac{m(b,s)}{s} \cdot \eta x > p(x) \right\}} \cdot \left(1 - \frac{m(b,s)}{s} \right) \cdot x + (1-a) \cdot x \right\} & \text{when } \delta = 1 \end{cases}
 \end{aligned}$$

As a result, when $\delta = 0$, $a^* = 1$, *i.e.*, investors with preference shock have to sell the claim of their projects. Investors with $\delta = 1$, however, could either participate in centralized or decentralized market ($a = 1$) or simply wait till $t = 2$ ($a = 0$). However, the above optimization implies that investors would never try centralized market due to search friction and bargaining.

First of all, competitive buyers set $p(x) = x$ in complete information. In this scenario, $p(x) > \frac{m(b,s)}{s} \eta x$ for all sellers- $(x, \delta = 0)$ and thus they trade in centralized market. Moreover, sellers- $(x, \delta = 1)$ would be indifferent between selling in centralized market at $t = 1$ and waiting till $t = 2$.

Secondly, in the presence of information asymmetry, $p(x) = p$ for all sellers pooling in centralized market. On one hand, for sellers with $\delta = 0$, if decentralized market does not exist, their only choice is the centralized market. If the decentralized market exists, however, they would compare $\frac{m(b,s)}{s} \eta x$ with p . Furthermore, if $\frac{m(b,s)}{s} \eta x_1 > p$, we would also have $\frac{m(b,s)}{s} \eta x_2 > p$ provided $x_2 > x_1$. Thus there may exist a cut-off point \tilde{x} on the choice of trading venues. If $\tilde{x} \in (x_L, x_H)$, then $\frac{m(b,s)}{s} \eta \tilde{x} = p$ holds by definition. On the other hand, for sellers with $\delta = 1$, as argued above, they would never consider trading in decentralized market even though it would be available. Instead, they simply compare p and x . As a result, those with $x < p$ would sell their asset claims in the centralized market at $t = 1$ while those with $x \geq p$ would enter either markets and wait till $t = 2$.

Finally, based the above two pieces of observation, we have

$$\begin{aligned}
U^s(x, \delta) &= \begin{cases} p & \text{if } \delta = 0 \text{ and } x \leq \tilde{x} \\ \frac{x}{\tilde{x}} \cdot p & \text{if } \delta = 0 \text{ and } x > \tilde{x} \\ p & \text{if } \delta = 1 \text{ and } x \leq p \\ x & \text{if } \delta = 1 \text{ and } x > p \end{cases} \\
&= \begin{cases} \max\{\frac{x}{\tilde{x}}, 1\} \cdot p & \text{if } \delta = 0 \\ \max\{\frac{x}{p}, 1\} \cdot p & \text{if } \delta = 1 \end{cases} \\
&= \max\left\{\frac{x}{p + (\tilde{x} - p) \cdot \mathbf{1}_{\{\delta=0\}}}, 1\right\} \cdot p
\end{aligned}$$

□

Proof of Corollary 1

Proof. It is immediately obtained by using Proposition 1 and 2.

□

Proof of Lemma 1

Proof. The results in the general case is proved as below.

First of all, we show that $p \leq \tilde{x}$. Eq. (2.4) suggests that

$$\begin{aligned}
p &= \frac{\pi F(\tilde{x}) \mathbb{E}(x|x \leq \tilde{x}) + (1 - \pi) F(p) \mathbb{E}(x|x \leq p)}{\pi F(\tilde{x}) + (1 - \pi) F(p)} \\
&= \frac{\pi \int_{x_L}^{\tilde{x}} x dF(x) + (1 - \pi) \int_{x_L}^p x dF(x)}{\pi F(\tilde{x}) + (1 - \pi) F(p)} \\
&\leq \frac{\pi \int_{x_L}^{\tilde{x}} \tilde{x} dF(x) + (1 - \pi) \int_{x_L}^p p dF(x)}{\pi F(\tilde{x}) + (1 - \pi) F(p)} \\
&= \frac{\pi \tilde{x} F(\tilde{x}) + (1 - \pi) p F(p)}{\pi F(\tilde{x}) + (1 - \pi) F(p)},
\end{aligned}$$

where the inequality strictly holds iff $x > x_L$. Thus $p \leq \frac{\pi \tilde{x} F(\tilde{x}) + (1 - \pi) p F(p)}{\pi F(\tilde{x}) + (1 - \pi) F(p)}$. Multiplying both side of this inequality with $\pi F(\tilde{x}) + (1 - \pi) F(p)$ and rearranging then yields $p \leq \tilde{x}$, where the equality holds iff $\tilde{x} = x_L (= p)$.

Secondly, Eq. (2.4) can be rewritten as

$$G(p, \tilde{x}, \pi) \equiv \pi \int_{x_L}^{\tilde{x}} x dF(x) + (1 - \pi) \int_{x_L}^p x dF(x) - \pi p F(\tilde{x}) - (1 - \pi) p F(p) = 0.$$

Thus we have

$$\begin{aligned} G_p \equiv \frac{\partial G}{\partial p} &= -[\pi F(\tilde{x}) + (1 - \pi)F(p)] < 0 \\ G_{\tilde{x}} \equiv \frac{\partial G}{\partial \tilde{x}} &= \pi(\tilde{x} - p)f(\tilde{x}) > 0 \end{aligned}$$

According to Implicit Function Theorem, we have

$$\frac{dp}{d\tilde{x}} = -\frac{G_{\tilde{x}}}{G_p} > 0.$$

Thus we can denote the above result as $p = P_{AS}(\tilde{x}, \pi)$, which is an increasing function of \tilde{x} . Furthermore, since $\tilde{x} \geq x_L$, we immediately have $p \geq x_L$. When $\tilde{x} = x_L$, Eq. (2.2) is reduced as follows.

$$p = \frac{\int_{x_L}^p x dF(x)}{F(p)} = \mathbb{E}(x|x \leq p),$$

which is a classic problem of adverse selection by Akerlof (1970) and the unique solution is $p = x_L$. As a result, $\mathcal{P}_{AS}(\tilde{x} = x_L) = x_L$ and thus $p \geq x_L$. So far we finish the proof that $x_L \leq p \leq \tilde{x}$, where both inequality strictly holds if $\tilde{x} > x_L$.

Moreover, we have

$$\frac{\partial G}{\partial \pi} = [\int_{x_L}^{\tilde{x}} x dF(x) - pF(\tilde{x})] - [\int_{x_L}^p x dF(x) - pF(p)]$$

Define $H(a; p) \equiv \int_{x_L}^a x dF(x) - pF(a)$. Then we have $\frac{\partial H}{\partial a} = (a - p)f(a)$ and thus $H(a; p)$ increases with a when $a > p$. Since $\tilde{x} > p$, we have

$$G_{\pi} \equiv \frac{\partial G}{\partial \pi} = H(\tilde{x}; p) - H(p; p) > 0,$$

which in turn, by using Implicit Function Theorem again, implies that

$$\frac{dp}{d\pi} = -\frac{G_{\pi}}{G_p} > 0.$$

Denote $p = P_{AS}(\tilde{x}, \pi)$. Thus $p = P_{AS}(\tilde{x}, \pi) \leq P_{AS}(\tilde{x}, \pi = 1) = \mathbb{E}(x|x \leq \tilde{x}) \leq \mathbb{E}(x|x \leq x_H) = \mu(\theta)$.

Finally, when $\pi = 0$, Eq. (2.4) is reduced to

$$p = \frac{\int_{x_L}^p x dF(x)}{F(p)} = \mathbb{E}(x|x \leq p),$$

which has been discussed above in the case when $\tilde{x} = x_L$. The only solution is $p = P_{AS}(\tilde{x}, \pi = 0) = x_L$ and CM totally collapses.

Now we prove the second part of Proposition 3.

When $x \stackrel{U}{\sim} X = [x_L, x_H]$, we have

$$F(x) = \begin{cases} 0 & \text{if } x < x_L \\ \frac{x-x_L}{x_H-x_L} & \text{if } x_L \leq x \leq x_H \\ 1 & \text{if } x > x_H \end{cases}.$$

Substituting $F(x)$ into Eq. (2.2) and making some algebraic manipulation yields Eq. (2.3). □

Proof of Lemma 2

Proof. The results in the general case are proved as follows. For the ease of argument, we list Eq. (2.4) and Eq. (2.3) as below.

$$\begin{aligned} m(1, \alpha)r(\tilde{x}) &= \frac{\kappa}{1-\eta} \\ m(\frac{1}{\alpha}, 1)\tilde{x} &= \frac{p}{\eta} \end{aligned}$$

In the above simultaneous equations, \tilde{x} and α are endogenous variables while κ , p and η are exogenous. To prove $\frac{\partial \tilde{x}}{\partial p} > 0$, we differentiate both sides of the above two equations. Then we have

$$\begin{pmatrix} m_2(1, \alpha)r(\tilde{x}) & m(1, \alpha)r'(\tilde{x}) \\ -\frac{m_1(\frac{1}{\alpha}, 1)}{\alpha^2}\tilde{x} & m(\frac{1}{\alpha}, 1) \end{pmatrix} \begin{pmatrix} \frac{d\alpha}{dp} \\ \frac{d\tilde{x}}{dp} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\eta} \end{pmatrix} \quad (\#)$$

Since $m_1 > 0$, $m_2 > 0$ and $r'(\tilde{x}) > 0$, Cramer rule immediately suggests that

$$\frac{d\tilde{x}}{dp} = \frac{\det \begin{pmatrix} m_2(1, \alpha)r(\tilde{x}) & 0 \\ -\frac{m_1(\frac{1}{\alpha}, 1)}{\alpha^2}\tilde{x} & \frac{1}{\eta} \end{pmatrix}}{\det \begin{pmatrix} m_2(1, \alpha)r(\tilde{x}) & m(1, \alpha)r'(\tilde{x}) \\ -\frac{m_1(\frac{1}{\alpha}, 1)}{\alpha^2}\tilde{x} & m(\frac{1}{\alpha}, 1) \end{pmatrix}} > 0 \text{ and } \frac{d\alpha}{dp} = \frac{\det \begin{pmatrix} 0 & m(1, \alpha)r'(\tilde{x}) \\ \frac{1}{\eta} & m(\frac{1}{\alpha}, 1) \end{pmatrix}}{\det \begin{pmatrix} m_2(1, \alpha)r(\tilde{x}) & m(1, \alpha)r'(\tilde{x}) \\ -\frac{m_1(\frac{1}{\alpha}, 1)}{\alpha^2}\tilde{x} & m(\frac{1}{\alpha}, 1) \end{pmatrix}} < 0 \quad (*)$$

Following the same strategy delivers that $\frac{d\tilde{x}}{d\kappa} > 0$ and $\frac{d\tilde{x}}{d\eta} < 0$.

Now we prove the second part of Lemma 2. It is immediately done with the assumption $x \stackrel{U}{\sim} X = [x_L, x_H]$ and $m(b, s) = \lambda \cdot \min\{b, s\}$.

□

Proof of Proposition 3

Proof. We proceed with the strategy of guess-and-verify. Assume market co-existence can be supported, *i.e.*, $\tilde{x} \in (x_L, x_H)$, then we have

$$\begin{aligned} \frac{\lambda \cdot \min\{b, s\}}{b} (1 - \eta) \mathbb{E}(x|x \geq \tilde{x}) &= \kappa \\ \frac{\lambda \cdot \min\{b, s\}}{s} \eta \tilde{x} &= p \\ s &= \pi \cdot [1 - F(\tilde{x})] \\ p &= \varphi \tilde{x} + (1 - \varphi) x_L \end{aligned}$$

Additionally, we assume that $s < b$, *i.e.*, $\alpha \equiv \frac{s}{b} < 1$, then the above equations suggest that $\tilde{x} = (\frac{1-\varphi}{\lambda\eta-\varphi})x_L$. We now have to check whether the guess that $\alpha < 1$ is valid. It then is easy for us to check that $\alpha < 1$ when $\kappa \leq \underline{\kappa}$. Moreover, we have to guarantee that $\tilde{x} = (\frac{1-\varphi}{\lambda\eta-\varphi})x_L \in (x_L, x_H)$, which is true if and only $\lambda\eta > \varphi$ and $\sigma > \hat{\sigma}$.

Similarly, we assume $\tilde{x} \in (x_L, x_H)$, but $\alpha \geq 1$. Then we can show this guess is true if and only $\lambda\eta > \varphi$ and $\sigma > \hat{\sigma}$ and $\kappa \in (\underline{\kappa}, \bar{\kappa})$.

Finally, based on the above analysis, $\tilde{x} = x_H$ would be the only equilibrium result if $\kappa \geq \bar{\kappa}$ when $\lambda\eta > \varphi$ and $\sigma > \hat{\sigma}$, or, for all $\kappa \in \mathbb{R}_+$, we have $\lambda\eta \leq \varphi$ and $\sigma \leq \hat{\sigma}$.

□

Proof of Proposition 4

Proof. First, based on Proposition 3, we have market co-existence both before and after government intervention. Moreover, since $\kappa_g = \kappa' < \kappa_b = \kappa$, we know from Proposition 3 that $\tilde{x}(\kappa_g) \leq \tilde{x}(\kappa_b)$ and thus $p(\kappa_g) \leq p(\kappa_b)$. Additionally, the decrease of information cost implies a more favorable extensive margin for sellers. Therefore, we know that

$$U(\tilde{x}(\kappa_g)) = p(\kappa_g) \leq p(\kappa_b) = \frac{m(s(\kappa_b), b(\kappa_b))}{s(\kappa_b)} \eta \tilde{x}(\kappa_b) \leq \frac{m(s(\kappa_g), b(\kappa_g))}{s(\kappa_g)} \eta \tilde{x}(\kappa_b) = U(\tilde{x}(\kappa_b)).$$

Since $U(x)$ increases with x , there exists a cut-off point $\hat{x} \in (\tilde{x}(\kappa_g), \tilde{x}(\kappa_b))$ such

that

$$U(x) \begin{cases} \leq p(\kappa_b) & \text{if } x \leq \hat{x} \\ \geq p(\kappa_b) & \text{if } x \geq \hat{x} \end{cases}.$$

□

2.6.2 Extension: Robustness Check

In our baseline model, we use random search to characterize search frictions in DM. Besides, we assume liquidity shock, δ , only adopts two mass points, zero and one. Thus buyers could *infer* only sellers with $\delta = 0$ could show up in DM. We use this appendix to undertake two pieces of robust check. First, we revisit the model with directed search rather than random search in DM. That is, each buyer in DM only run submarket- x , where sellers- (x, δ) would meet buyers. Secondly, we treat the general case on liquidity shock, in which δ is continuously distributed over an interval, just like the asset payoff x . In the general case, buyers can only detect asset payoff x with costly information acquisition, but have idea on liquidity shock δ . Thus buyers in DM would instead launch optimal contract to extract true value of δ for those sellers self-selecting into DM. Finally, we have so far focused on an exchange economy, *i.e.*, all of potential sellers is exogenous endowed with one unit of asset at $t = 0$.

Directed Search

In direct search, each buyer with information investment only engage in certain submarket- x . Correspondingly, seller- (x, δ) directly trade with buyers there. Since in equilibrium buyers would be indifferent among different sub-markets in DM, the following equation holds for all seller- (x, δ) self-selecting into DM.

$$\frac{m(b(x), s(x))}{b(x)}(1 - \eta)x = \kappa, \quad (2.13)$$

which immediately implies that $\alpha(x) \equiv \frac{s(x)}{b(x)}$, the market tightness at submarket- x , increases with x . In turn, the expected revenue of entering DM by seller- (x, δ) , $\frac{m(b(x), s(x))}{s(x)}\eta x$ increases with x , just as that in random search. As a result, the property of cut-off point on market participation is preserved in the case with directed search. Following the pro-

cedure in Section 2 and 3, we have the following results on equilibrium choice of trading venues with directed search.

Corollary 11. (Equilibrium Choice of Trading Venues) Denote $\underline{\kappa}' \equiv \lambda(1-\eta)(\frac{1-\varphi}{\lambda\eta-\varphi})x_L$, $\underline{\kappa} \equiv \frac{\lambda(1-\eta)}{2}[(\frac{1-\varphi}{\lambda\eta-\varphi})x_L + x_H]$ and $\bar{\kappa} \equiv \lambda(1-\eta)x_H$.

1. (Choice of Trading Venues)

- (a) For sellers with $\delta = 0$, there exists a cut-off point $\tilde{x}' \in [x_L, x_H]$ such that if $x \geq \tilde{x}'$, they would self-select into DM, and enter CM otherwise at $t = 1$.
- (b) For sellers with $\delta = 1$, if $x < p$, they would choose CM, and if $x \geq p$, they would participate in neither DM nor CM at $t = 1$, but instead wait to consume at $t = 2$.

2. (Characterization of Cut-off Value \tilde{x}')

- (a) When $\lambda\eta > \varphi$ and $\sigma > \hat{\sigma} \equiv (\frac{1-\lambda\eta}{1+\lambda\eta-2\varphi})\mu$ (i.e., $\lambda\eta > \varphi + (1-\varphi)\frac{x_L}{x_H}$), where $\varphi = \varphi(\pi) \equiv \frac{\sqrt{\pi}}{\sqrt{\pi}+1}$, we have $\underline{\kappa}' < \underline{\kappa} < \bar{\kappa}$ and

$$\begin{aligned}\tilde{x}' &= \min\{x_H, \max\{(\frac{1-\varphi}{\lambda\eta-\varphi}) \cdot x_L, \frac{\kappa}{\lambda(1-\eta)}\}\} \\ &= \begin{cases} x_H & \text{if } \kappa > \bar{\kappa} \\ \frac{\kappa}{\lambda(1-\eta)} & \text{if } \underline{\kappa}' < \kappa \leq \bar{\kappa} \cdot \\ (\frac{1-\varphi}{\lambda\eta-\varphi})x_L & \text{if } \kappa \leq \underline{\kappa}' \end{cases} \\ p &= \varphi(\pi) \cdot \tilde{x}' + (1-\varphi(\pi)) \cdot x_L.\end{aligned}$$

- (b) When $\lambda\eta \leq \varphi(\pi)$ or $\sigma \leq \hat{\sigma}$ (i.e., $\lambda\eta \leq \varphi + (1-\varphi)\frac{x_L}{x_H}$), we have

$$\tilde{x} = x_H \text{ for all } \kappa \in \mathbb{R}_+.$$

Proof. Since $m(b(x), s(x)) = \lambda \cdot \min\{b(x), s(x)\}$, Eq. (2.13) suggests that, to recover information investment, informed buyers in DM would only accept to trade with sellers with $x \geq \frac{\kappa}{\lambda(1-\eta)}$. Then following the proof strategy of Proposition 5 yields the desired results. □

This corollary implies the main results in the benchmark with random search are still preserved in our robust check with directed search. We illustrate key results of this

corollary in Figure 2.9. Two comments are made here. First of all, the pattern of venue choice is qualitatively the same between random and directed search. Secondly, $\tilde{x}' \leq \tilde{x}$, *i.e.*, the size of DM tends to be smaller under directed search than that under random search. As suggested in Section 4, a smaller DM would be save more social resources. Therefore our result is complementary to the findings by Moen (1997) on efficiency obtained by directed search.

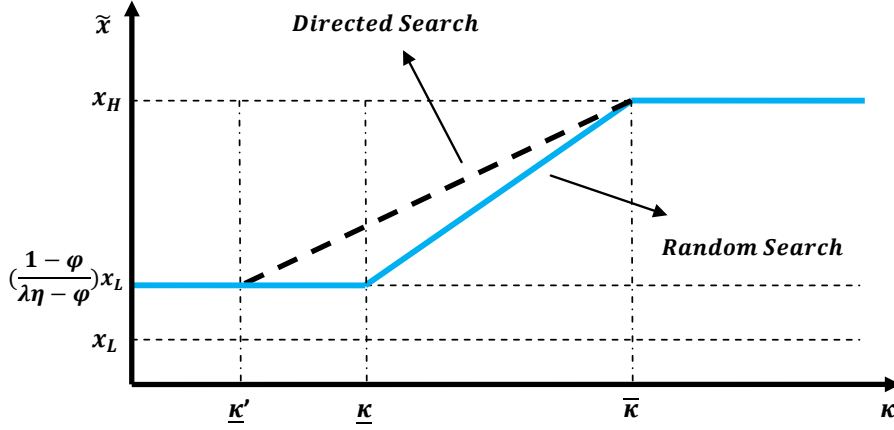


Figure 2.9: Choice of Trading Venues: Direct vs Random Search

Optimal Contract with a Continuum of δ

Now we return to random search with matching function $m(b, s) = \lambda \cdot \min\{b, s\}$. The second line of model extension on switching from $\delta \in \{0, 1\}$ to $\delta \in [0, 1]$. Notice that buyers can no longer infer the true value of δ . Now we assign all bargaining power to buyers in DM and they could initiate optimal contract $\{q(x, \delta), \tau(x, \delta)\}$ after paying information cost κ in DM. Given any x , $q(x, \delta)$ and $\tau(x, \delta)$ denote the quantity of asset transferred to buyers and the consumption paid to sellers respectively if sellers report his type of private value as δ . Note that x is verifiable after buyers incurring information cost κ .

In the same spirit in the benchmark, there exists a cut-off value of κ , say $\bar{\kappa}'$, above which DM cannot not be supported whatever the contract buyers propose in DM. In contrast, the equilibrium with market co-existence is not only sustainable, but also stable if $\kappa < \bar{\kappa}^*$. Moreover, there exists another cut-off point $\underline{\kappa}^* < \bar{\kappa}^*$ such that $b < s$ in equilibrium if $\kappa < \underline{\kappa}^*$. Since the co-existence of CM and DM is the most intriguing part, we assume

$\kappa < \bar{\kappa}^*$ holds. Moreover, to focus on the characterization of optimal contract by buyers in DM, we assume $\kappa < \underline{\kappa}^* < \bar{\kappa}^*$ throughout this subsection such that DM can not only be supported, but also there are more buyers than sellers flowing into DM. We can prove that the qualitative results shown below are still held if $\kappa \in \underline{\kappa}^* < \bar{\kappa}^*$ (and $0 < b < s$ correspondingly).

Denote $U(x, \delta)$ as the gain of seller- (x, δ) by enrolling in the contract by buyers in DM. Now seller- (x, δ) makes her discrete choice among three alternatives.

$$\max \left\{ \underbrace{p}_{CM}, \underbrace{\frac{m(b, s)}{s} \cdot U(x, \delta) + \left[1 - \frac{m(b, s)}{s}\right] \cdot \delta x}_{DM}, \underbrace{\delta x}_{No-trade} \right\}.$$

In this part, we focus on the most intriguing case, *i.e.*, the co-existence between CM and DM. Thanks to Revelation Principle, given any x , we could simply focus on buyer's direct mechanism in DM, which is formulated as below.

$$\Pi_B(x) \equiv \max_{\{q(x, \delta) \in [0, 1], \tau(x, \delta) \in [0, \infty)\} \mathbf{z}_{DM|x}} \left\{ \int_{\delta \in \mathbf{Z}_{DM|x}} [-\tau(x, \delta) + q(x, \delta) \cdot x] \right\}$$

subject to

$$\begin{aligned} U(x, \delta) &\equiv U_x(\delta; \delta) = \max_{\delta' \in \mathbf{Z}_{DM|x}} \{U_x(\delta; \delta')\} \\ U_x(\delta; \delta') &\equiv [1 - q(x, \delta')] \cdot \delta x + \tau(x, \delta') \quad (IC) \\ \frac{m(b, s)}{s} \cdot U(x, \delta) + \left[1 - \frac{m(b, s)}{s}\right] \cdot \delta x &\geq \underline{\mathbf{U}}(\mathbf{p}, \delta \mathbf{x}) \equiv \mathbf{max}\{\mathbf{p}, \delta \mathbf{x}\} \quad (IR). \end{aligned}$$

Similar to standard mechanism design, both Incentive Compatibility (*IC*) and Individual Rationality (*IR*) should be satisfied. What makes our setup challenging is that, buyer's mechanism design would affect \mathbf{Z}_{DM} , the content of seller- (x, δ) self-selecting into the contracts in DM. Moreover, it is worth noting the outside option is type-dependent and thus may involve in the so-called countervailing incentive *a la* Lewis and Sappington (1989), Maggi and Rodriguez-Clare (1995) and Jullien (2000).

The following lemma on seller's choice of trading venues generalizes the results of Proposition 2.

Lemma 6. (Seller's Choice of Trading Venues) For any seller- (x, δ) , given p in CM and contract $\{q(x, \delta), \tau(x, \delta)\}$ in DM, there exists cut-off values $\underline{\delta}(x)$ and $\bar{\delta}(x)$ such that,

1. if $\delta \in [0, \underline{\delta}(x)]$, she sells her asset at CM;
2. if $\delta \in (\underline{\delta}(x), \bar{\delta}(x)]$, she sells her asset at DM;

3. if $\delta \in (\bar{\delta}(x), 1]$, she chooses not to trade.

Proof. We rewrite the *IR* condition of this mechanism design as below.

$$U(x, \delta) \geq V(x, \delta, p) \equiv \delta x + \frac{1}{\hat{\lambda}} \max\{p - \delta x, 0\},$$

where $\hat{\lambda} \equiv \frac{m(b,s)}{s}$ and thus $V(x, \delta, p)$ decreases with δ when $\delta x < p$ while increases when $\delta x > p$. The non-monotone property of V stems from the fact that, relative to DM, seller- (δ, x) have two outside options. One is sell at CM at price p while the other one is pure autarky, *i.e.*, participating in neither CM nor DM. When δx is low, the outside option with CM is larger than that in with autarky. It is just opposite when δx is high enough.

Secondly, given any x , Envelope Theorem suggests that

$$\frac{\partial U(x, \delta)}{\partial \delta} = [1 - q(x, \delta)]x \geq 0.$$

Combining these two observations yields the results in the lemma with

$$\begin{aligned} \underline{\delta}(x) &= \max\{0, \frac{p - \hat{\lambda} \cdot U(x, \delta)}{(1 - \hat{\lambda}) \cdot x}\} \\ \bar{\delta}(x) &= \min\{1, \frac{U(x, \delta)}{x}\} \end{aligned}$$

Finally, a figure with δ in horizontal axis and U, V in vertical axis would help illustrate our findings. Due to space concern, we omit it here. □

Based on the above lemma, we reach the optimal contract in DM and in turn obtain the explicit solution on $(\underline{\delta}(x), \bar{\delta}(x))$.

Proposition 8. (*Optimal Contract Offered in DM*) When $\delta \stackrel{U}{\sim} \Delta = [0, 1]$ and $m(b, s) = \lambda \cdot \min\{b, s\}$ and κ is small enough, given any x in DM, buyer's optimal contract is in the form as take-it-or-leave-it in the following form.

$$\{q^*(x, \delta) = 1, \tau^*(x, \delta) = \tau(x)\},$$

where

$$\tau(x) = \begin{cases} \frac{p+x}{2} & \text{if } x \in [p, \frac{2-\lambda}{\lambda} \cdot p] \\ \frac{p}{\lambda} & \text{if } x \in (\frac{2-\lambda}{\lambda} \cdot p, \frac{2}{\lambda} \cdot p] \\ \frac{x}{2} & \text{if } x \in (\frac{2}{\lambda} \cdot p, x_H] \end{cases}$$

In turn, we have

$$\underline{\delta}(x) = \max\left\{0, \frac{p - \lambda \cdot \tau(x)}{(1 - \lambda) \cdot x}\right\} = \begin{cases} 1 & \text{if } x \in [x_L, p] \\ \frac{(1 - \frac{\lambda}{2})p - \frac{\lambda}{2}x}{(1 - \lambda)x} & \text{if } x \in (p, \frac{2 - \lambda}{\lambda} \cdot p] \\ 0 & \text{if } x \in (\frac{2 - \lambda}{\lambda} \cdot p, \frac{2}{\lambda} \cdot p] \\ 0 & \text{if } x \in (\frac{2}{\lambda} \cdot p, x_H] \end{cases}.$$

$$\bar{\delta}(x) = \min\left\{1, \frac{\tau(x)}{x}\right\} = \begin{cases} 1 & \text{if } x \in [x_L, p] \\ \frac{p + x}{2x} & \text{if } x \in (p, \frac{2 - \lambda}{\lambda} \cdot p] \\ \frac{p}{\lambda x} & \text{if } x \in (\frac{2 - \lambda}{\lambda} \cdot p, \frac{2}{\lambda} \cdot p] \\ \frac{1}{2} & \text{if } x \in (\frac{2}{\lambda} \cdot p, x_H] \end{cases}.$$

Proof. The first part is proved as below.

To ease illustration while preserving the key insights, we have assumed that information cost κ is low enough such that $b > s$ is always true in equilibrium. In turn, we have $\frac{m(b,s)}{s} = \lambda \in (0, 1)$, a constant. This would help us focus on characterizing optimal contract by buyers. We break down the proof into the following steps.

First of all, since sellers could always sell their assets at price p in CM and $\delta \in [0, 1]$, buyers in DM would have no customers if $U(x, \delta) < p$. Meanwhile, to recover information cost, buyers in DM ex ante would never accept sellers with $x < p$.

Secondly, for seller- (x, δ) self-selecting into DM and is allowed to trade with buyers there, denote $\tilde{\delta}(x) = \min\{\frac{p}{x}, 1\} = \frac{p}{x}$. Since Then we can check that $\underline{\delta}(x) \leq \tilde{\delta}(x) \leq \bar{\delta}(x)$, where $\underline{\delta}(x)$ and $\bar{\delta}(x)$ are characterized in the proof of Lemma 1. For each x , buyers launch direct mechanism for two groups respectively. One is $\delta \in \Delta_1 = [\underline{\delta}(x), \tilde{\delta}(x)]$ while the other group is $\delta \in \Delta_2 = [\tilde{\delta}(x), \bar{\delta}(x)]$. On one hand, for each group, buyers make sure *IR* and *IC* conditions are satisfied. On the other hand, buyers have to make sure sellers in group $\Delta_1 \cup \Delta_2$ would have no incentive to deviate the other group. After all, even though x is verifiable after buyers incur information cost, δ is still un-observable. As a result, incentive compatibility of not deviating to another group has to be additionally taken into account. In the next two pieces of analysis, we first solve the within-group contract and then go to discussion of *IC* on across-group.

Buyer's objective function for group Δ_1 is

$$\Pi_B(x)|_{\Delta_1} \equiv \max_{\{q(x, \delta) \in [0, 1], \tau(x, \delta) \in [0, \infty)\}} \left\{ \int_{\underline{\delta}(x)}^{\tilde{\delta}(x)} [-\tau(x, \delta) + q(x, \delta) \cdot x] \right\}$$

Meanwhile, for group with $\delta \in \Delta_1$, the outside option is simplified as $V(x, \delta, p) \equiv \delta x + \frac{1}{\lambda} \max\{p - \delta x, 0\} = \frac{p}{\lambda} - (\frac{1}{\lambda} - 1)\delta x$. That is, for sellers in this group, the outside option decreases with δ . Following Maggi and Rodriguez-Clare (1995), among others, we define $\Upsilon(x, \delta) = U(x, \delta) - V(x, \delta, p)$. Envelope Theorem suggests

$$\frac{\partial \Upsilon}{\partial \delta} = [\frac{1}{\lambda} - q(x, \delta)] \cdot x.$$

Thus

$$[1 - q(x, \delta)]\delta x + \tau(x, \delta) - [\frac{p}{\lambda} - (\frac{1}{\lambda} - 1)\delta x] = \int_{\underline{\delta}(x)}^{\delta} [\frac{1}{\lambda} - q(x, \delta')] x d\delta'.$$

Expressing the above equation for $\tau(x, \delta)$ and substituting it into the buyer's objective function for group Δ_1 mentioned above, we can easily prove that, for group Δ_1 , $q^*(x, \delta)|_{\Delta_1} = 1$. Substituting it into the above equation suggests that $\tau^*(x, \delta)|_{\Delta_1}$ has nothing to do with δ and is thus denoted as $\tau^*(x)|_{\Delta_1}$.

Similarly, we can show that $q^*(x, \delta)|_{\Delta_2} = 1$ and $\tau^*(x, \delta)|_{\Delta_2}$ also has nothing to do with δ and is thus denoted as $\tau^*(x)|_{\Delta_2}$. Finally, to make sure the *IC* condition of across-group is satisfied, we have to make sure $\tau^*(x, \delta)|_{\Delta_1} = \tau^*(x, \delta)|_{\Delta_2} \equiv \tau(x)|_{\Delta_1 \cup \Delta_2} = \tau(x)$. In sum, given $x > p$ and buyers and sellers are matched in DM, the optimal contract would take the form as $\{q^*(x, \delta) = 1, \tau^*(x, \delta) = \tau(x)\}$. It is obvious that $\tau(x) \leq x$ is always held.

In turn, we have $U(x, \delta) = \tau(x)$ and thus

$$\begin{aligned}\underline{\delta}(x) &= \max\{0, \frac{p - \lambda \cdot \tau(x)}{(1 - \lambda) \cdot x}\} \\ \bar{\delta}(x) &= \min\{1, \frac{\tau(x)}{x}\} = \frac{\tau(x)}{x}.\end{aligned}$$

As a recap, buyer's profit function focusing on sellers with x is

$$\Pi_B(x) \equiv \max_{\{q(x, \delta) \in [0, 1], \tau(x, \delta) \in [0, \infty)\} \mathbf{z}_{DM|x}} \left\{ \int_{\delta \in \mathbf{z}_{DM|x}} [-\tau(x, \delta) + q(x, \delta) \cdot x] \right\}$$

Using the optimal contract and cut-off values just obtained above, $\Pi_B(x)$ is refined as below.

$$\Pi_B(x) = \max_{\tau(x, \delta) \in [0, x]} [x - \tau(x)] [G(\bar{\delta}(x)) - G(\underline{\delta}(x))]$$

subject to

$$\begin{aligned}\underline{\delta}(x) &= \max\{0, \frac{p - \lambda \cdot \tau(x)}{(1 - \lambda) \cdot x}\} \\ \bar{\delta}(x) &= \min\{1, \frac{\tau(x)}{x}\} = \frac{\tau(x)}{x},\end{aligned}$$

where G denotes the CDF of δ with support $[0, 1]$. If we further assume $\delta \stackrel{U}{\sim} \Delta = [0, 1]$, then we obtain $\tau(x)$ as that in Proposition 5. In turn, we obtain $\underline{\delta}(x)$ and $\bar{\delta}(x)$ as in the second part of this proposition. We are done. \square

We illustrate Proposition 5 in Figure 2.10 and 2.11 respectively the terms of trade by buyers and choice of trading venues by sellers.

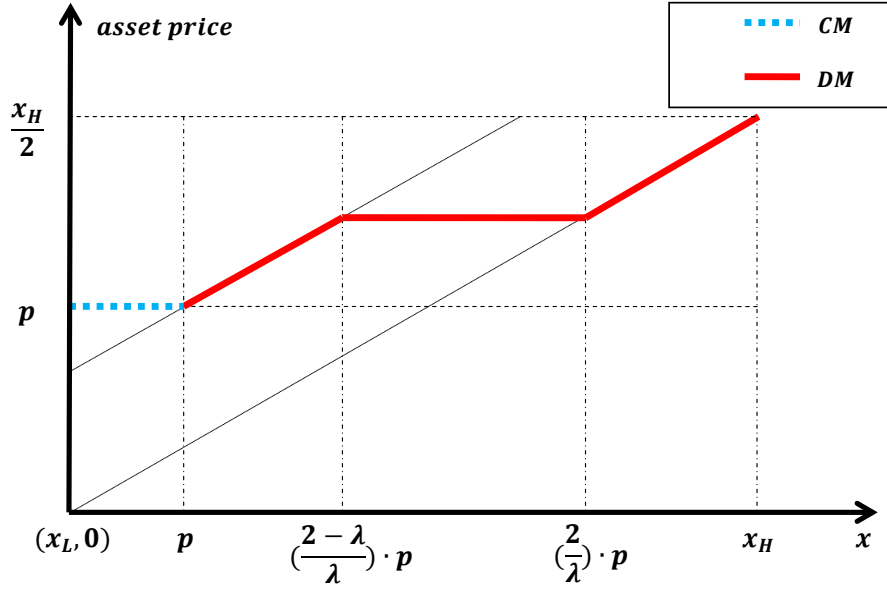


Figure 2.10: $(p, \tau(x))$: Assets Prices in CM and DM

Several remarks are made here. First of all, the optimal contract by buyers in DM only focus on sellers with $x > p$. On one hand, p is always an outside option of any seller- (x, δ) and thus buyers in DM would attract no sellers if $\tau(x, \delta) < p$. On the other hand, buyer's profit is $x - \tau(x, \delta)$. To make the profit non-negative, it must that they would trade with $x > p$ and x can always be verifiable. Secondly, given price in CM p , seller's choice over CM, DM and autarky not depends on common value x , but also on private value δ . Thirdly, we are still in the position of partial equilibrium since price in CM is taken as given. Based on Proposition 7, p is solved in equilibrium as below.

$$p = \frac{\int_{x_L}^p x dF(x) + \int_p^{\min\{\frac{2-\lambda}{\lambda} \cdot p, x_H\}} x G\left(\frac{(1-\frac{\lambda}{2})p - \frac{\lambda}{2}x}{(1-\lambda)x}\right) dF(x)}{F(p) + \int_p^{\min\{\frac{2-\lambda}{\lambda} \cdot p, x_H\}} G\left(\frac{(1-\frac{\lambda}{2})p - \frac{\lambda}{2}x}{(1-\lambda)x}\right) dF(x)}, \quad (2.14)$$

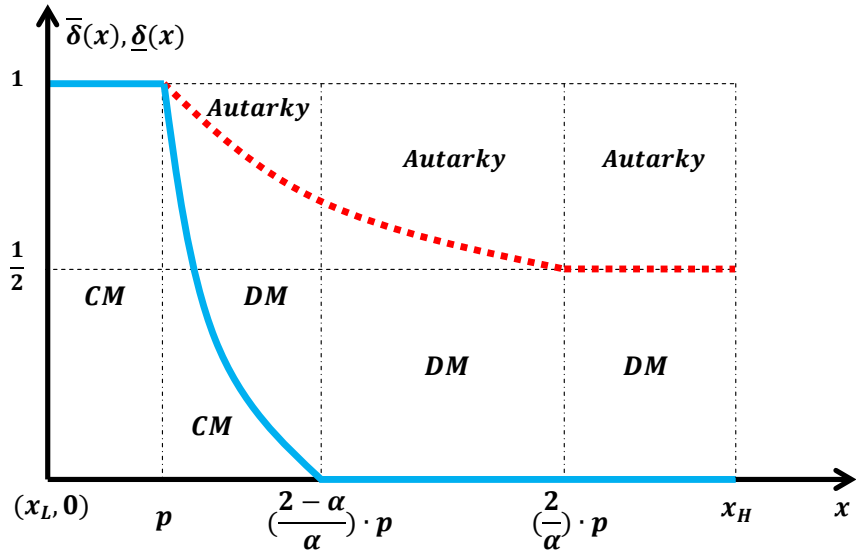


Figure 2.11: Seller's Choice of Trading Venues

where F and G denotes the CDF of x and δ respectively. Moreover, we have implicitly assumed $G(\delta)$ is a uniform distribution. However, even though $F(x)$ is a uniform distribution, the above equation has no explicit solution on p .

Chapter 3

A Search-Based Theory of The Life-Cycle Pattern of Asset Holding

3.1 Introduction

The vast majority of financial assets, such as corporate bonds, US federal funds and mortgage-backed securities, are typically traded in decentralized markets. As documented by Harris (2003), Duffie, Gârleanu and Pedersen (2005, DGP thereafter) and Duffie (2012), decentralized markets, which are sometimes called over-the-counter markets (OTC), are mainly characterized with search frictions and bilateral bargaining.¹ Liquidity mis-allocation emerges since it takes time for buyers and sellers to be matched with each other for the trading surplus.²

All of the literature on asset search assumes an infinite horizon environment, which in turn contributes to tractability. Moreover, as shown by Storesletten, Telmer and Yaron (2004), Chambers and Schlagenhauf (2003), and Chang and Hong (2012),

¹Not only for financial assets, it is also true that non-financial durable goods, such as used aircraft, are also traded with search frictions. See Gavazza (2011) for details.

²There may exist more fundamental reasons than just search frictions. For example, as suggested by Wasmer and Weil (2004), imperfections in labor and credit markets may stem from moral hazard or adverse selection. For simplicity, we throughout this paper sticks to search friction as a convenient reduced form rather than assuming any information asymmetries.

investment on financial assets, including stocks and bonds, is hump-shaped over one's life. Motivated by the empirical regularity, we investigate the implications of search frictions for the life cycle as well as for the aggregate distribution of investor's asset holdings. Based on the model, we further address the effect of search friction on asset mis-allocation in terms of both cross section and time series. We fully characterize not only the stationary distribution, but also the transitional dynamics of asset holding. More importantly, we obtain analytic results for the life cycle of asset holdings by each cohort. To the best of our knowledge, this paper is the first to analytically characterize the life-cycle pattern of asset trading.

The model developed in this paper has several pieces of testable implications. First of all, the stationary size distribution of asset holding follows a logarithmic pattern. Secondly, the life cycle of asset holding by each cohort conforms to a geometric distribution while the size distribution of asset holding in each cross-section follows a logarithmic pattern. Thirdly, the average growth rate of asset holding is irrelevant to the size of current asset holdings. Meanwhile, the volatility of growth rate of asset holding decreases with the size of current asset holdings. That is, we reach the results on Gibrat's law on asset trading in OTC.³

This paper is mostly related to DGP (2005) and Lagos and Rocheteau (2009). The former considers asset trading by risk-neutral investors in OTC with two types of preference (low and high) and strict restriction on asset holding (zero or one unit of asset). The latter uses quasi-linear utility to model general types of preference and relaxes the assumption on asset indivisibility. There are several key differences and connections between these two works and this paper. First of all, DGP (2005) considers only two types of preference and asset holdings are assumed to be either zero or one unit. In contrast, our paper allows general types of preference and investors could accumulate any countable units of assets. Secondly, DGP (2005) focuses on steady state while our paper takes in account both stationary equilibrium and transitional dynamics. It is true that Lagos and Rocheteau (2009) already proposes a tractable model to admit asset divisibility, transitional dynamics and

³See Sutton (1997) and Klette and Kortum (2004), among other, for the detailed description of Gibrat's law for firm dynamics.

general types of preference. As shown in Lagos and Rocheteau (2007), one of the key limits in Lagos and Rocheteau (2009) is that, when there are only two types of preference as in DGP (2005), there are only four mass points for the equilibrium size of asset holdings.⁴ In contrast, we not only consider the general preference, but also show that, even with only two types of preference, there are countably infinite types for the size distribution of asset holdings. Thirdly, both DGP (2005) and Lagos and Rocheteau (2009) address a closed system, *i.e.*, investors always live in the financial market to re-balance their asset holdings from time to time. As a result, our paper complements to these two works by using birth-and-death process in firm dynamics to model the implication of search friction for the life-cycle pattern of asset holding. Finally, we incorporate into our model the empirical findings of fire-sale and fire-purchase by Coval and Stafford (2007). These empirical features are absent in both works.

The issues our paper addresses belong to the literature on asset search while the modeling strategy originates from the literature on firm dynamics. Early works consists of Lucas (1978), Jovanovic (1982), Hopenhayn (1992) and Ericson and Pakes (1995) among others. Recent research includes Cooley and Quadrini (2001), Cabral and Mata (2003), Klette and Kortum (2004) and Luttmer (2007, 2011). The methodology our paper mainly adopts from the field of firm dynamics is stochastic process of birth and death. That is, the modeling block of this paper essentially stems from Klette and Kortum (2004) and Luttmer (2011).

The rest of this paper proceeds as below. Section 2 describes the environment of asset trading in OTC. Section 3 formulates the dynamic system and pins down the stationary equilibrium. Section 4 fully characterizes the non-stationary life-cycle pattern of asset holding by each cohort of investors. It also offers analytic solutions to transitional dynamics of the distribution of asset holdings. Section 5 investigates the implication of search frictions for several dimensions of asset liquidity. Section 6 explores several pieces of model extension. Section 7 concludes. All proofs are pooled in the Appendix.

⁴In general, as shown in Lagos and Rocheteau (2009), if investors have $I \in \mathbb{N}$ types of preference, there would be I^2 pieces of masses points in equilibrium for the size distribution of asset holdings.

3.2 Environment

Time proceeds continuously from zero to infinity. There are two kinds of risk-neutral agents in the economy: investors and market-makers (dealers). There is one kind of non-depreciable and non-reproducible asset circulating in this exchange economy.⁵ We use $s \in \mathbb{R}_{++}$ to denote the fixed supply of this asset. Each unit of the asset constantly delivers x units of fruits.

Every investor has a risk-free bank account, in which she can deposit her real money balance with the interest rate $r \in \mathbb{R}_{++}$, the same as her discount rate. Additionally, investors in the financial market may be hit by preference shock, *i.e.*, they may switch between low-preference (L) and high-preference (H).⁶ Heterogeneity on preference types may be due to investor's heterogeneous financing cost or ability in managing assets. Preference shock is assumed to be public information.⁷ Similar to DGP (2005) and Lagos and Rocheteau (2009), we specify investor's preference directly over their asset holding. We use x_L and x_H to denote the valuation of one unit of asset by low-preference and high-preference investors respectively. Without loss of generality, we assume $0 < x_L < x_H = x$.

All trade is assumed to be dealer-intermediated. There is no short sell. Without loss of generality, investors with any preference type are allowed to hold any countable units of assets.⁸ Each unit of asset is randomly matched with dealer with an independent Poisson process, which governs the amounts of assets investors hold. In particular, dealers encounter each asset as buyers and sellers with Poisson rate λ_i^- and λ_i^+ respectively, where $i \in \{L, H\}$ denotes the preference type of investors. See

⁵The asset could be treated as either stock or bond. I don't distinguish them in the model.

⁶We analyze the case with general types of preference in the section on model extension.

⁷We focus on the effect of search friction on life cycle of asset trading and size distribution of asset holding. Thus we assume there is no information asymmetry on the public or private values of assets. Discussion on adverse selection of asset quality in OTC includes Guerrerri, Shimer and Wright (2010), Chiu and Koepl (2011), Guerrerri and Shimer (2012a,b) and Chang (2012). Zhang (2012), on the other hand, considers liquidity mis-allocation in OTC due to private values on asset.

⁸Lagos and Rocheteau (2009) and Gârleanu (2009) imposes no restriction on the amount of asset holding. However, in equilibrium, there are only countable (actually finite) levels of asset holdings.

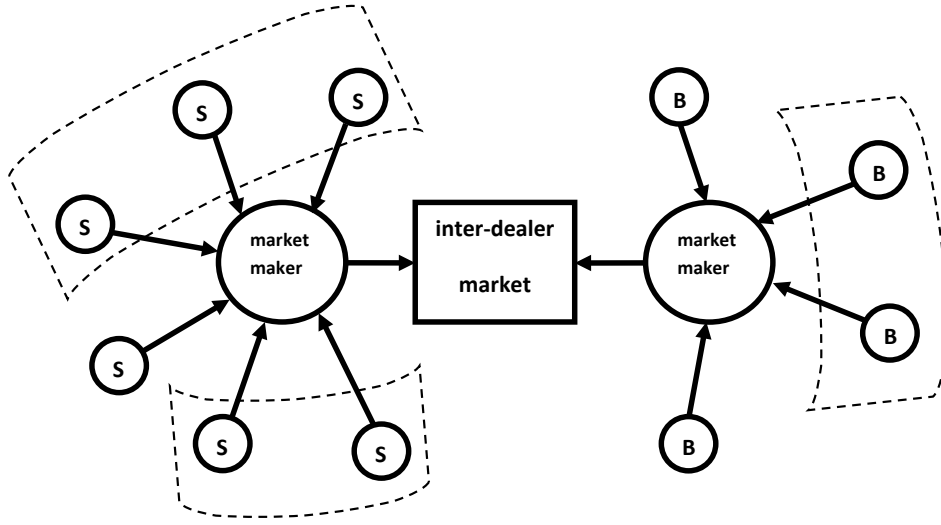


Figure 3.1: **An Example on the Snapshot of Trading Structures for Investors (Asset Sellers and Buyers) and Market Makers. Some investors have multiple units of assets while the others have one unit.**

Figure 3.1 for illustration of trading in certain snapshot of time. Complementing to the cross-section illustration on asset trading in OTC, Figure 3.2 presents the dynamics of asset trading as well as entry and exiting the financial market. Intuitively, low-preference investors desire to sell while high-preference ones to buy assets. Moreover, to accommodate the empirical findings by Coval and Stafford (2007), we allow the possibility of fire-purchase and fire-sale by low-preference and high-preference investors respectively. In sum, the Poisson rate of selling and buying one unit of assets by investor with type- i preference are $n \cdot \lambda_i^-$ and $n \cdot \lambda_i^+$ respectively, where $i \in \{L, H\}$.⁹

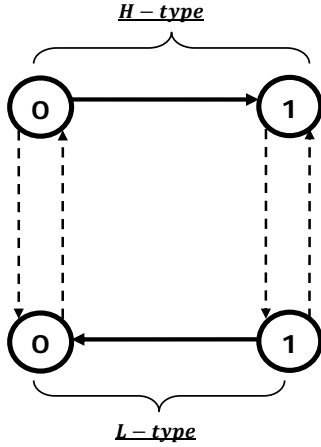
Search intensity is exogenously given in the baseline. On one hand, investors are interpreted to exit the market if she sells the last unit of her asset holdings. On the other hand, potential investors can enter OTC by paying certain fixed cost and then buying one unit of assets. In our baseline, the measure of market-makers is exogenously given and normalized to be one. To keep the model under control, we follow the assumption made in DGP (2005) and Lagos and Rocheteau (2009) that market-makers never hold asset inventory. Market-makers manage to do so by

⁹See Morris (1998), among others, for the additive property of independent Poisson process.

having access to immediate inter-dealer market.¹⁰

In the next two sections, we first address each investor's value function and the size distribution of asset holding in steady state. Then we switch to the discussion on individual's life cycle of asset trading. In the end, we characterize the transitional dynamics on the size distribution of asset holdings.

(A): *Duffie, Gârleanu and Pedersen (2005)*



(B): *This Paper*

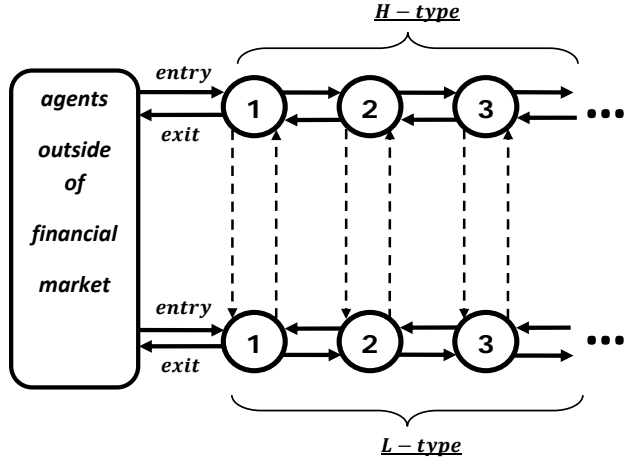


Figure 3.2: Comparison of Model Structure between DGP (2005) and This Paper.

3.3 Steady State

3.3.1 Value Function

In our baseline model, investors have two states on preference, *i.e.*, $i(t) \in \{L, H\}$. Denote W_t as the current wealth in her risk-less bank account with interest rate $r > 0$. Intuitively, high-preference investors would like to buy while low-preference ones to sell. Correspondingly, we denote A_t the ask price at which high-type investors buy from market-makers, B_t the bid price at which low-type investors sell to market-makers in normal time. Moreover, we denote $P_t^{\text{fire-purchase}}$ the price at which low-type

¹⁰Weill (2007) and Lagos, Rocheteau and Weill (2011) extend DGP (2005) and Lagos and Rocheteau (2009) by considering the possibility of inventory by dealers.

investors buy from market-makers due to fire purchase and $P_t^{\text{fire-sale}}$ the price at which high-type investors sell to market-makers due to the shock of fire sale.¹¹ Then the value function of a investor- $\{i(t), n_t, W_t\}$ is formulated as below.

$$V(i(t), n_t, W_t) = \sup_{C, n} \mathbb{E}_t \int_0^\infty e^{-r\tau} dC_{t+\tau}$$

subject to

$$dW_t = rW_t dt - dC_t + n_t(x - \delta \mathbf{1}_{\{i(t)=L\}})dt - \tilde{P}_t dn_t$$

where \mathbb{E}_t denotes \mathcal{F}_t -conditional expectation, C_t is a cumulative process on consumption, $n_t \in \mathbb{N}$ is a feasible process on asset holdings, ξ^n is the type process induced by n , and at the time t of a trading event, $\tilde{P}_t \in \{A_t, B_t, P_t^{\text{fire-sale}}, P_t^{\text{fire-purchase}}\}$ is the trade price, which depends on the type of counterparty. There are triple state variables $\{i, n, W\}$. For notational ease, we use $V_i(n, W)$ for $V(i, n, W)$ throughout the rest of the paper. Then we obtain the following value functions in steady state.¹²

$$\begin{aligned} rV_L(n, W) &= n \cdot x_L + n\lambda_L^- \cdot [V_L(n-1, W+B) - V_L(n, W)] + n\lambda_L^+ \cdot [V_L(n+1, W - P^{\text{fire-purchase}}) - V_L(n, W)] \\ &\quad + \lambda_u \cdot [V_H(n, W) - V_L(n, W)] + \frac{\partial V_L(n, W)}{\partial W} \cdot rW \\ rV_H(n, W) &= n \cdot x_H + n\lambda_H^+ \cdot [V_H(n+1, W-A) - V_H(n, W)] + n\lambda_H^- \cdot [V_H(n-1, W + P^{\text{fire-sale}}) - V_H(n, W)] \\ &\quad - \lambda_d \cdot [V_H(n, W) - V_L(n, W)] + \frac{\partial V_H(n, W)}{\partial W} \cdot rW \end{aligned}$$

First of all, when low-preference and high-preference investors have the opportunity to be matched with dealers to sell and buy their asset, it is a mutually beneficially process. Following the standard way in search literature, we use Nash bargaining to split trading surplus between both parties. In particular, assume bargaining power of market-makers and investors is $z \in (0, 1)$ and $1 - z$ respectively. Denote M as the price prevalent in the inter-dealer market. Then bid-price B and ask-price A are determined as below.

$$\begin{aligned} B(n, W) &\in \underset{B \geq 0}{\operatorname{argmax}} [V_L(n-1, W+B) - V_L(n, W)]^{1-z} [M - B]^z \\ A(n, W) &\in \underset{A \geq 0}{\operatorname{argmax}} [V_H(n+1, W-A) - V_H(n, W)]^{1-z} [A - M]^z \end{aligned}$$

¹¹See Coval and Stafford (2007) for empirical document on fire-sale and fire-purchase.

¹²See Dixit and Pindyck (1994) or the appendix of DGP (2005), among others, for the deriving strategies on value functions with continuous-time setup.

Secondly, investors suffer loss when the negative shocks, *i.e.*, shocks on fire-sale or fire-purchase, knocks the door. There are multiple ways to determine $P^{fire-purchase}$ and $P^{fire-sale}$. For the ease of expression and calculation, we simply assume that dealers makes zero profit and investors incur loss when they are matched in the presence of negative shock.¹³ That is,

$$P_t^{fire-purchase} = P^{fire-sale} = M.$$

Finally, assume asset price in the inter-dealer market is determined by

$$M \in \underset{M' \geq 0}{argmax} \left\{ \sum_{n=1}^{\infty} [V_L(n-1, W+B) - V_L(n, W) + M-B] \cdot \mu_L^n \right\}^\epsilon \cdot \left\{ \sum_{n=1}^{\infty} [V_H(n+1, W-A) - V_H(n, W) + A-M] \cdot d\mu_H^n \right\}^{1-\epsilon},$$

where μ_i^n denotes the measure of type- i agents with n units of assets at time t , $i \in \{L, H\}$, $n \in \mathbb{N}$ while ϵ denotes the bargaining power of buyer-side.

To make it tractable for us to obtain analytic results on the size distribution of asset holding, we make the following restrictions on $\{\lambda_L^+, \lambda_L^-, \lambda_H^+, \lambda_H^-\}$.

Assumption 4. $\theta \equiv \frac{\lambda_L^+}{\lambda_L^-} = \frac{\lambda_H^+}{\lambda_H^-} \in (0, 1)$

Moreover, we make the following assumption as a sufficient condition to make Proposition 1 hold.

Assumption 5. $0 < z < 1 - \theta$.

Proposition 9. (Value Function and Bid-Ask Spread)

1. For investors with preference-type $i \in \{L, H\}$, n units of financial assets and W units of liquid assets, the value function is the following form.

$$V_i(n, W) = v_i \cdot n + W,$$

¹³We can check that other forms of specification would not change the results qualitatively. For example, we could set $P^{fire-purchase} = v_H$ and $P^{fire-sale} = v_L$, in which dealers make positive profits while investors suffer more.

where

$$\begin{aligned}
v_H - v_L &= \frac{x_H - x_L}{r + \lambda_u + \lambda_d + (1 - \epsilon)[\lambda_H^- - \lambda_H^+(1 - z)] + \epsilon[\lambda_L^-(1 - z) - \lambda_L^+]} > 0 \\
v_L &= \frac{x_L}{r} + \{\epsilon[\lambda_L^-(1 - z) - \lambda_L^+] + \lambda_u\} \cdot \frac{v_H - v_L}{r} \in (\frac{x_L}{r}, v_H) \\
v_H &= \frac{x_H}{r} - \{(1 - \epsilon)[\lambda_H^- - \lambda_H^+(1 - z)] + \lambda_d\} \cdot \frac{v_H - v_L}{r} \in (v_L, \frac{x_H}{r})
\end{aligned}$$

2. The bid and ask prices, the spread and asset price of the inter-dealer market are obtained as below.

$$\begin{aligned}
B(n, W) &= v_L + (1 - z)\epsilon(v_H - v_L) \in (v_L, A) \\
A(n, W) &= v_H - (1 - z)(1 - \epsilon)(v_H - v_L) \in (B, v_H) \\
Spread &\equiv A - B = z \cdot (v_H - v_L) \\
M &= \epsilon v_L + (1 - \epsilon)v_H \in (A, B)
\end{aligned}$$

Two pieces of comments are made here. First of all, according to Proposition 1, even though there is no information asymmetry as in Glosten and Milgrom (1985), the bid-ask spread still emerges in the presence of search frictions $(\lambda_L^+, \lambda_L^-, \lambda_H^+, \lambda_H^- < \infty)$, heterogeneity in preference types $(x_H \neq x_L)$ and positive bargaining power of dealers $(z > 0)$. Secondly, notice that similar analytic results are also obtained in DGP (2005). Thus another key message of this proposition is that, even removing the restriction on asset holdings, we could still have very neat and intuitive results for the effects of search friction on bid-ask spreads.

3.3.2 Free Entry of New Investors into Financial Markets

The market participants makes the following decision on entering financial markets. On one hand, if they does not participate in the market, then given their real money balance \tilde{m} , the value function in steady state is characterized by

$$rU_{NOT}(\tilde{W}) = \frac{\partial U_{NOT}(\tilde{W})}{\partial \tilde{W}} r\tilde{W}.$$

On the other hand, if they choose to enter the market, then

$$rU_{IN}^i(\widetilde{W}) = -\kappa_i\eta_i + \eta_i[V_i(1, \widetilde{W}) - U_{IN}^i(\widetilde{W})] + \frac{\partial U_{IN}^i(\widetilde{W})}{\partial \widetilde{W}} r\widetilde{W} \text{ for } i \in \{L, H\}$$

In equilibrium, we have $U_{NOT}(\widetilde{W}) = U_{IN}^H(\widetilde{W}) = U_{IN}^L(\widetilde{W})$ for any $\widetilde{W} \in \mathbb{R}$. Thus $\kappa_i = V_i(1, \widetilde{W}) - U_{IN}^i(\widetilde{W})$ for $i \in \{L, H\}$. The entry rate $\{\eta_L, \eta_H\}$ is not pinned down here. Instead, it would be determined using other conditions to be shown in the next sub-section.

3.3.3 Distribution of Asset Holding in Steady State

Denote $\mu_i^n(t)$ as the measure of type- i agents with n units of assets at time t , where $i \in \{L, H\}$, $n \in \mathbb{N}$ and $t \in \mathbb{R}_+$. Denote $\mu_i^{entry}(t)$ as the measure of investors newly entering the financial markets at t . For simplicity, each new-entry investor starts with one unit of asset.

Corresponding to Figure 3.2, the dynamics on $\mu_i^n(t)$ is formulated as below.

$$\begin{aligned} d\mu_L^n(t)/dt &= \begin{cases} (n+1)\lambda_L^- \cdot \mu_L^{n+1}(t) - n(\lambda_L^- + \lambda_L^+) \cdot \mu_L^n(t) + (n-1)\lambda_L^+ \cdot \mu_L^{n-1}(t) \\ + \lambda_d \cdot \mu_H^n(t) - \lambda_u \cdot \mu_L^n(t) & \text{when } n \in \mathbb{N} \setminus \{1\} \\ (n+1)\lambda_L^- \cdot \mu_L^{n+1}(t) - n(\lambda_L^- + \lambda_L^+) \cdot \mu_L^n(t) \\ + \lambda_d \cdot \mu_H^n(t) - \lambda_u \cdot \mu_L^n(t) + \mu_L^{entry}(t) \cdot \eta_L(t) & \text{when } n = 1 \end{cases} \\ d\mu_H^n(t)/dt &= \begin{cases} (n+1)\lambda_H^- \cdot \mu_H^{n+1}(t) - n(\lambda_H^+ + \lambda_H^-) \cdot \mu_H^n(t) + (n-1)\lambda_H^+ \cdot \mu_H^{n-1}(t) \\ - \lambda_d \cdot \mu_H^n(t) + \lambda_u \cdot \mu_L^n(t) & \text{when } n \in \mathbb{N} \setminus \{1\} \\ (n+1)\lambda_H^- \cdot \mu_H^{n+1}(t) - n(\lambda_H^+ + \lambda_H^-) \cdot \mu_H^n(t) \\ - \lambda_d \cdot \mu_H^n(t) + \lambda_u \cdot \mu_L^n(t) + \mu_H^{entry}(t) \cdot \eta_H(t) & \text{when } n = 1 \end{cases} \end{aligned}$$

Taking $d\mu_L^n(t)/dt$ with $n \geq 2$ for example, the first and the third items denote the inflow from investors with the same preference type while the second one the outflow to investors with the same preference type. Meanwhile, the fourth and the fifth items are the inflow and outflow due to preference shocks.

The stability of the dynamic system requires outflow be equal to inflow, *i.e.*, for $i \in \{L, H\}$ and $t \in \mathbb{R}_+$,

$$\mu_i^{entry}(t)\eta_i(t) = \lambda_i^- \mu_i^1(t).$$

Then the total measure of sellers and buyers are constant over time. That is, the

total population $\mu(t) \equiv \mu_L(t) + \mu_H(t) \equiv \sum_{n=1}^{+\infty} \mu_L^n(t) + \sum_{n=1}^{+\infty} \mu_H^n(t)$ is constant, which is to be determined. WLOG, we normalize such that $\mu_L^{entry}(t) = \mu_H^{entry}(t) = 1$ for all $t \in \mathbb{R}_+$.

Additionally, the market clearing condition is formulated as below.

$$\sum_{n=1}^{+\infty} \lambda_L^- n \mu_L^n(t) + \sum_{n=1}^{+\infty} \lambda_H^- n \mu_H^n(t) = \sum_{n=1}^{+\infty} \lambda_H^+ n \mu_H^n(t) + \sum_{n=1}^{+\infty} \lambda_L^+ n \mu_L^n(t) + \sum_{i \in \{L, H\}} \mu_i^{entry}(t) \eta_i(t) \quad (3.1)$$

$$\sum_{n=1}^{+\infty} n \mu_L^n(t) + \sum_{n=1}^{+\infty} n \mu_H^n(t) = s \quad (3.2)$$

where s denotes the total amount of assets circulating in the economy. On one hand, Eq. (3.1) states that, since dealers are assumed to never hold no inventory, the amount of assets investors sell to dealers (left-hand side) must equal to that dealers sell to investors (right-hand side). Eq. (3.2), on the other hand, requires that at each point the aggregate demand of assets by investors in OTC must be equal to the fixed supply.

In steady state, $d\mu_i^n(t)/dt = 0$ for all $i \in \{L, H\}$ and $n \in \mathbb{N}$. In turn, we reach Proposition 1.

Proposition 10. We formulate the distribution of asset holding in steady state as well as the Gibrat's law for asset holding as below.

1. **(Stationary Size Distribution of Asset Holding)** The size distribution of asset holding conforms to a logarithmic distribution with parameter θ , *i.e.*,

$$\mu_i^n = \frac{\mu_i^1 \cdot \theta^{n-1}}{n} \text{ for } i \in \{L, H\} \text{ and } n \in \mathbb{N}.$$

where $\mu_L^1 = s(1 - \theta)\pi_L$, $\mu_H^1 = (\frac{\lambda_u}{\lambda_d}) \cdot \mu_L^1 = s(1 - \theta)\pi_H$, $\pi_L \equiv \frac{\lambda_d}{\lambda_u + \lambda_d}$, $\pi_H \equiv \frac{\lambda_u}{\lambda_u + \lambda_d}$, and the total population of investors in the economy is

$$\mu \equiv \sum_{n=1}^{+\infty} (\mu_L^n + \mu_H^n) = \frac{s(1 - \theta)}{\theta} \cdot \ln\left(\frac{1}{1 - \theta}\right).$$

Moreover, the *endogenous* entry rate is

$$\eta_L = \lambda_L^- \mu_L^1 = s(1 - \theta)\lambda_L^- \pi_L.$$

$$\eta_H = \lambda_H^- \mu_H^1 = s(1 - \theta)\lambda_H^- \pi_H$$

2. **(Gibrat's Law)** The average growth rate of asset holding is independent of investor's size while the volatility of growth rate decreases with the size of asset holdings, *i.e.*, for $i \in \{L, H\}$, we have

$$\begin{aligned}\mathbb{E} \left[\frac{dn_i(t)/dt}{n_i(t)} \right] &= \lambda_i^+ - \lambda_i^- \\ \text{Var} \left[\frac{dn_i(t)/dt}{n_i(t)} \right] &= \frac{\lambda_i^+ + \lambda_i^-}{n_i(t)}\end{aligned}$$

Based on the proposition, for $i \in \{L, H\}$ and $n \in \mathbb{N}$, the probability distribution, $\mathcal{M}_i^n \equiv \frac{\mu_i^n}{\mu}$, and the total probability, $\mathcal{M}^n \equiv \mathcal{M}_L^n + \mathcal{M}_H^n$, are immediately obtained as below.

$$\mathcal{M}_L^n = \frac{\pi_L \cdot \theta^n}{n \cdot \ln(\frac{1}{1-\theta})}, \quad \mathcal{M}_H^n = \frac{\pi_H \cdot \theta^n}{n \cdot \ln(\frac{1}{1-\theta})}, \quad \mathcal{M}^n = \frac{\theta^n}{n \cdot \ln(\frac{1}{1-\theta})} \text{ for } n \in \mathbb{N}.$$

and the probability of both types is

$$\mathcal{M}_L \equiv \sum_{n=1}^{+\infty} \mathcal{M}_L^n = \pi_L, \quad \mathcal{M}_H \equiv \sum_{n=1}^{+\infty} \mathcal{M}_H^n = \pi_H,$$

We illustrate the (truncated) probability distribution of asset holding in Figure

3.3.

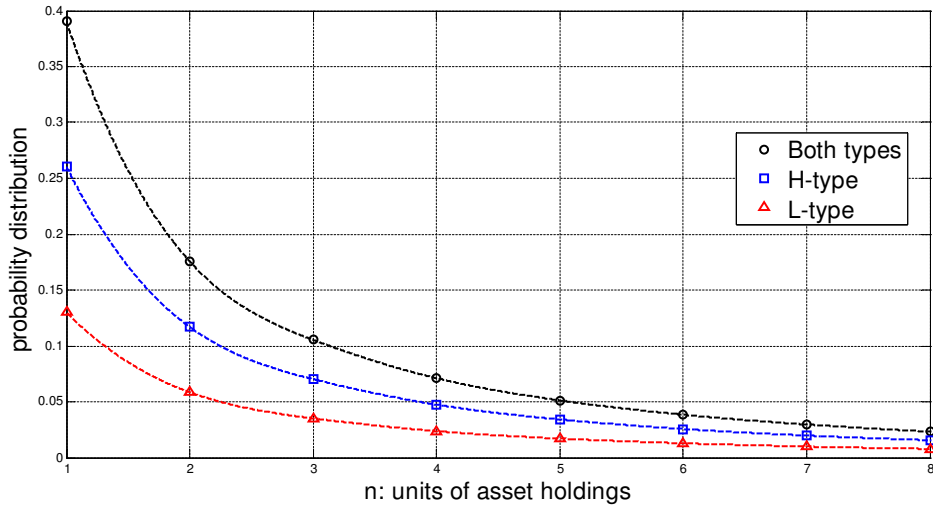


Figure 3.3: **The Size Distribution of Asset Holdings and Its Decomposition in Steady State (parameter values: $\lambda_u/\lambda_d = 2$ and $\theta = 0.9$.)**

3.4 Transition Dynamics

The previous section mainly focuses on the stationary distribution of asset holdings in each cross section. This section advances to the analysis on non-stationary life-cycle pattern of each cohort. Let $p_i^n(t; n_0)$ denote the probability that certain investor with i -type preference has n units of asset holdings at time t given that she has n_0 units at time 0. In contrast to the dynamics on the aggregate size distribution of asset holding in Section 3.3, by definition there is no new entry into each cohort. More specifically, the associated dynamics associated with the (in-truncated) probability of investors is formulated as below.

$$\begin{aligned}
 dp_L^n(t; n_0)/dt &= \begin{cases} (n+1)\lambda_L^- \cdot p_L^{n+1}(t; n_0) - n(\lambda_L^- + \lambda_L^+) \cdot p_L^n(t; n_0) + (n-1)\lambda_L^+ \cdot p_L^{n-1}(t; n_0) \\ + \lambda_d \cdot p_H^n(t; n_0) - \lambda_u \cdot p_L^n(t; n_0) & \text{when } n \in \mathbb{N} \setminus \{1\} \\ (n+1)\lambda_L^- \cdot p_L^{n+1}(t; n_0) - n(\lambda_L^- + \lambda_L^+) \cdot p_L^n(t; n_0) + \lambda_d \cdot p_H^n(t; n_0) - \lambda_u \cdot p_L^n(t; n_0) & \text{when } n = 1 \\ (n+1)\lambda_L^- \cdot p_L^{n+1}(t; n_0) & \text{when } n = 0 \end{cases} \\
 dp_H^n(t; n_0)/dt &= \begin{cases} (n+1)\lambda_H^- \cdot p_H^{n+1}(t; n_0) - n(\lambda_H^+ + \lambda_H^-) \cdot p_H^n(t; n_0) + (n-1)\lambda_H^+ \cdot p_H^{n-1}(t; n_0) \\ - \lambda_d p_H^n(t; n_0) + \lambda_u \cdot p_L^n(t; n_0) & \text{when } n \in \mathbb{N} \setminus \{1\} \\ (n+1)\lambda_H^- \cdot p_H^{n+1}(t; n_0) - n(\lambda_H^+ + \lambda_H^-) \cdot p_H^n(t; n_0) - \lambda_d \cdot p_H^n(t; n_0) + \lambda_u \cdot p_L^n(t; n_0) & \text{when } n = 1 \\ (n+1)\lambda_H^- \cdot p_H^{n+1}(t; n_0) & \text{when } n = 0 \end{cases}
 \end{aligned}$$

and by definition we have

$$\sum_{n=1}^{+\infty} p_H^n(t; n_0) + \sum_{n=0}^{+\infty} p_L^n(t; n_0) = 1.$$

Then we reach the following proposition.

Proposition 11. (Life Cycle of Asset Holding)

1. If we set the boundary conditions for both types as below.

$$p_L^1(0, n_0 = 1) = \pi_L \equiv \frac{\lambda_d}{\lambda_u + \lambda_d}, \quad p_H^1(0, n_0 = 1) = \pi_H \equiv \frac{\lambda_u}{\lambda_u + \lambda_d},$$

then we have

$$\begin{aligned}
p_L^0(t; n_0 = 1) &= \frac{\pi_L \lambda_L^- \cdot [1 - e^{-(\lambda_L^- - \lambda_L^+)t}]}{\lambda_L^- - \lambda_L^+ \cdot e^{-(\lambda_L^- - \lambda_L^+)t}} = \frac{\pi_L \cdot [1 - e^{-(1-\theta)\lambda_L^- t}]}{1 - \theta \cdot e^{-(1-\theta)\lambda_L^- t}} \\
p_L^1(t; n_0 = 1) &= [\pi_L - p_L^0(t; n_0 = 1)] \cdot [1 - \sigma(t)] \\
p_L^n(t; n_0 = 1) &= p_L^{n-1}(t; n_0 = 1) \cdot \sigma(t) \text{ for } n \in \mathbb{N} \setminus \{1\} \\
p_H^n(t; n_0 = 1) &= \left(\frac{\lambda_u}{\lambda_d}\right) \cdot p_L^n(t; n_0 = 1) \text{ for } n \in \mathbb{N} \cup \{0\} \\
p^n(t; n_0 = 1) &\equiv p_H^n(t; n_0 = 1) + p_L^n(t; n_0 = 1) = \frac{p_L^n(t; n_0 = 1)}{\pi_L} \text{ for } n \in \mathbb{N}
\end{aligned}$$

where $\sigma(t) \equiv \left(\frac{\lambda_L^+}{\lambda_L^-}\right) \cdot p_L^0(t; n_0 = 1) = \frac{\pi_L \cdot \theta \cdot [1 - e^{-(1-\theta)\lambda_L^- t}]}{1 - \theta \cdot e^{-(1-\theta)\lambda_L^- t}}$. Furthermore, at time t , conditioning on still staying in OTC, the (truncated) size distribution of investors with preference type i from the zero-cohort conforms to a geometric distribution as below.

$$\frac{p_i^n(t; n_0 = 1)}{1 - p_i^0(t; n_0 = 1)} = [1 - \sigma(t)]\sigma(t)^{n-1}, \text{ for } n \in \mathbb{N}.$$

and thus it also holds for the aggregate level $p^n(t; n_0 = 1)$.

2. If we remove the restriction on the boundary condition for investors with either preference, but instead assume that $\lambda_L^- = \lambda_H^- \equiv \lambda^-$ and $\lambda_L^+ = \lambda_H^+ \equiv \lambda^+ = \theta\lambda^-$, and $p^1(0, n_0 = 1) = 1$, then $p^n(t; n_0 = 1)$, the life-cycle evolution of both type, is formulated as below.

$$\begin{aligned}
p^0(t; n_0 = 1) &= \frac{\lambda^- \cdot [1 - e^{-(\lambda^- - \lambda^+)t}]}{\lambda^- - \lambda^+ \cdot e^{-(\lambda^- - \lambda^+)t}} = \frac{[1 - e^{-(1-\theta)\lambda^- t}]}{1 - \theta \cdot e^{-(1-\theta)\lambda^- t}} \\
p^1(t; n_0 = 1) &= [1 - p^0(t; n_0 = 1)] \cdot [1 - \sigma(t)] \\
p^n(t; n_0 = 1) &= p^{n-1}(t; n_0 = 1) \cdot \sigma(t) \text{ for } n \in \mathbb{N} \setminus \{1\},
\end{aligned}$$

where now $\sigma(t)$ in this case is adjusted to $\sigma(t) \equiv \left(\frac{\lambda^+}{\lambda^-}\right) \cdot p^0(t; n_0 = 1) = \frac{\theta[1 - e^{-(1-\theta)\lambda^- t}]}{1 - \theta \cdot e^{-(1-\theta)\lambda^- t}}$.

Moreover, the (truncated) size distribution of investors with n units of asset holding from the zero-cohort has a geometric distribution as in the first case.

The key differences between these two results lie in their assumption. In the first case, we have strong assumption on the initial condition for both types of investors,

and obtain the life-cycle pattern of asset holding for not only the aggregate level, but also for each type. In contrast, the second case relaxes the above restriction but instead only imposes assumption on the aggregate level. Then the analytic results still applies to the aggregate level, but the dynamics for either types of investors are typically unavailable.

Based on either case in the above proposition, we use Figure 3.4 to present the transitional dynamics of asset holdings by investors in the same cohort *at* $t = 0$.

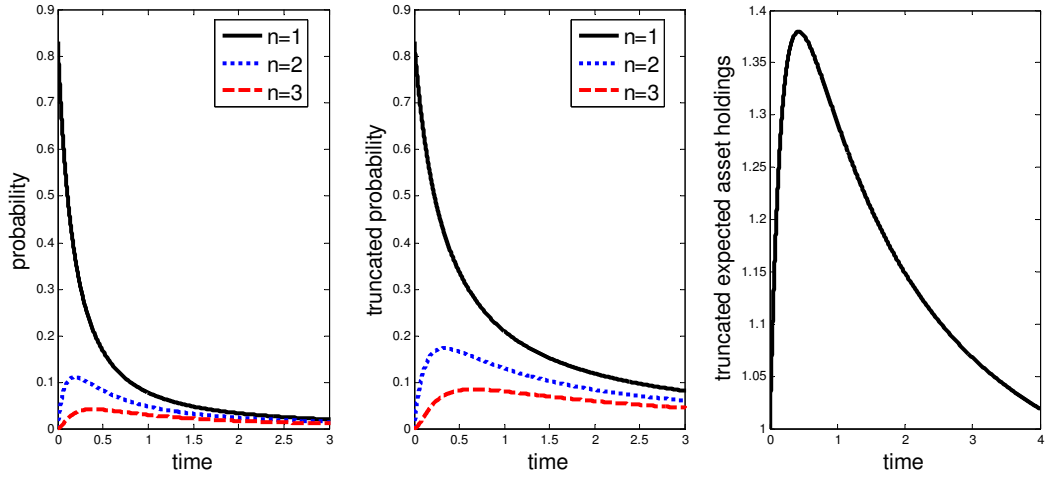


Figure 3.4: **Transitional Dynamics of Asset Holdings by Investors of Each Cohort**

Moreover, notice that $\frac{\eta_H}{\eta_H + \eta_L} = \frac{(\frac{\lambda_H^-}{\lambda_L^-}) \cdot (\frac{\lambda_u}{\lambda_d})}{(\frac{\lambda_H^-}{\lambda_L^-}) \cdot (\frac{\lambda_u}{\lambda_d}) + 1}$. If we additionally assume $\lambda_H^- = \lambda_L^-$, then $\frac{\eta_H}{\eta_H + \eta_L} = \bar{\pi}_H$ and thus the above transitional dynamics within one cohort also characterize the evolution of new entrants in the same cohort. [More comments here: to be completed.]

Finally, the above proposition immediately helps to reach the following result.

Corollary 12. (Expected Length of Trading in OTC)

$$\mathbb{E}(T_i) = \frac{\ln(\frac{\lambda_i^-}{\lambda_i^- - \lambda_i^+})}{\lambda_i^+} = \frac{\ln(\frac{1}{1-\theta})}{\lambda_i^+}.$$

Intuitively, when $\theta = \frac{\lambda^+}{\lambda^-} \in (0, 1)$ decreases, on average it takes less time for investors to sell out all of their assets and then exit the financial markets.

3.4.1 Preference Shock

With the same notations as before, we use $\mathcal{M}_i^n(t)$ to denote the proportion of investors with preference type $i \in \{L, H\}$ and holding n units of assets at time t . Then dynamics on $\mathcal{M}_i^n(t)$ is characterized as below.

$$\begin{aligned} d\mathcal{M}_L^n(t)/dt &= \begin{cases} (n+1)\lambda_L^- \cdot \mathcal{M}_L^{n+1}(t) - n(\lambda_L^- + \lambda_L^+) \cdot \mathcal{M}_L^n(t) + (n-1)\lambda_L^+ \cdot \mathcal{M}_L^{n-1}(t) \\ + \lambda_d \cdot \mathcal{M}_H^n(t) - \lambda_u \cdot \mathcal{M}_L^n(t) \end{cases} & \text{when } n \in \mathbb{N} \setminus \{1\} \\ (n+1)\lambda_L^- \cdot \mathcal{M}_L^{n+1}(t) - n\lambda_L^+ \cdot \mathcal{M}_L^n(t) + \lambda_d \cdot \mathcal{M}_H^n(t) - \lambda_u \cdot \mathcal{M}_L^n(t) & \text{when } n = 1 \\ d\mathcal{M}_H^n(t)/dt &= \begin{cases} (n+1)\lambda_H^- \cdot \mathcal{M}_H^{n+1}(t) - n(\lambda_H^+ + \lambda_H^-) \cdot \mathcal{M}_H^n(t) + (n-1)\lambda_H^+ \cdot \mathcal{M}_H^{n-1}(t) \\ - \lambda_d \cdot \mathcal{M}_H^n(t) + \lambda_u \cdot \mathcal{M}_L^n(t) \end{cases} & \text{when } n \in \mathbb{N} \setminus \{1\} \\ (n+1)\lambda_H^- \cdot \mathcal{M}_H^{n+1}(t) - n\lambda_H^+ \cdot \mathcal{M}_H^n(t) - \lambda_d \cdot \mathcal{M}_H^n(t) + \lambda_u \cdot \mathcal{M}_L^n(t) & \text{when } n = 1 \end{aligned}$$

Then we get to the following explicit solution on the transitional dynamics due to preference shock.

Proposition 12. (Dynamics by Preference Shock) Assume that we've been in the position of steady state by $t = 0-$, *i.e.*,

$$\mathcal{M}_L^n(t) = \frac{\pi_L \cdot \theta^n}{n \cdot \ln(\frac{1}{1-\theta})}, \quad \mathcal{M}_H^n(t) = \frac{\pi_H \cdot \theta^n}{n \cdot \ln(\frac{1}{1-\theta})} \text{ for } n \in \mathbb{N}.$$

Then, suddenly the proportion of low-preference investors increases, *i.e.*, $\mathcal{M}_L(t) \equiv \sum_{n=1}^{+\infty} \mathcal{M}_L^n(t)$ goes up from $\mathcal{M}_L(0-) = \pi_L$ to any $\mathcal{M}_L(0) \in (\pi_L, 1]$. Then the dynamics of $\{\mathcal{M}_i^n(t)\}_{i \in \{L, H\}, n \in \mathbb{N}}^{t \in \mathbb{R}_{++}}$ is characterized as follows.

$$\mathcal{M}_L^n(t) = \frac{\mathcal{M}_L(t) \cdot \theta^n}{n \cdot \ln(\frac{1}{1-\theta})}, \quad \mathcal{M}_H^n(t) = \frac{\mathcal{M}_H(t) \cdot \theta^n}{n \cdot \ln(\frac{1}{1-\theta})} \text{ for } n \in \mathbb{N} \text{ and } t \in \mathbb{R}_{++},$$

where

$$\begin{aligned} \mathcal{M}_L(t) &\equiv [1 - e^{-(\lambda_u + \lambda_d)t}] \cdot \pi_L + e^{-(\lambda_u + \lambda_d)t} \cdot \mathcal{M}_L(0) \\ \mathcal{M}_H(t) &\equiv [1 - e^{-(\lambda_u + \lambda_d)t}] \cdot \pi_H + e^{-(\lambda_u + \lambda_d)t} \cdot \mathcal{M}_H(0) \end{aligned}$$

As a result, $\{\mathcal{M}_L^n(t)\}_{t \in \mathbb{R}_{++}}$ decreases while $\{\mathcal{M}_H^n(t)\}_{t \in \mathbb{R}_{++}}$ increases over time with

$$\lim_{t \rightarrow \infty} \mathcal{M}_L^n(t) = \mathcal{M}_L^n, \quad \lim_{t \rightarrow \infty} \mathcal{M}_H^n(t) = \mathcal{M}_H^n.$$

Figure 3.5 shows the transitional dynamics due to preference shock on asset holdings.

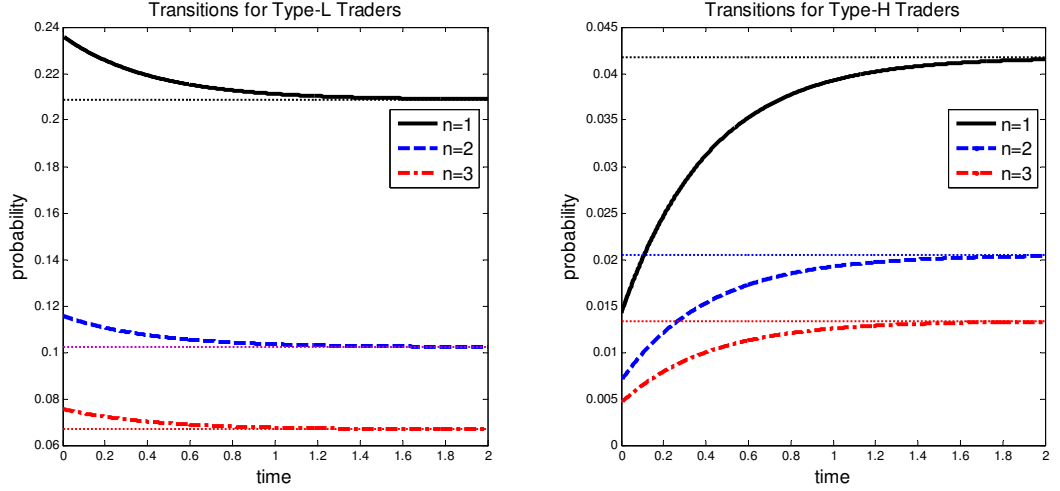


Figure 3.5: **Transitional Dynamics of Asset Holdings Due to Preference Shock;** Parameter values are to be added.

The dynamics of $\{\mu_i^n(t)\}_{i \in \{L,H\}, n \in \mathbb{N}}^{t \in \mathbb{R}_{++}}$ is immediately obtained as below.

$$\mu_i^n(t) = \frac{\mathcal{M}_i^n(t)}{\mu} \text{ for } n \in \mathbb{N} \text{ and } t \in \mathbb{R}_{++}$$

where $\mu \equiv \frac{s(1-\theta)}{\theta} \cdot \ln\left(\frac{1}{1-\theta}\right)$.

3.4.2 Redistributing Asset Holdings by Lessening Inequality

We are also in the position of dynamics system on $\{\mathcal{M}_i^n(t)\}_{i \in \{L,H\}, n \in \mathbb{N}}^{t \in \mathbb{R}_{++}}$. In addition to preference shock, another interesting departure from the stationary distribution is about redistributing asset holdings. One example is formulated as below.

Assume that we've been in the position of steady state by $t = 0-$, *i.e.*,

$$\mathcal{M}_L^n = \frac{\pi_L \cdot \theta^n}{n \cdot \ln(\frac{1}{1-\theta})}, \mathcal{M}_H^n = \frac{\pi_H \cdot \theta^n}{n \cdot \ln(\frac{1}{1-\theta})} \text{ for } n \in \mathbb{N}.$$

If we redistribute the asset holdings such that at $t = 0$, we have $\mathcal{M}_L^1(0) = \pi_L$ and $\mathcal{M}_H^1(0) = \pi_H$, *i.e.*, there is no inequality on asset holdings within each group $i \in \{L, H\}$. Then the dynamics of $\{\mathcal{M}_i^n(t)\}_{i \in \{L, H\}, n \in \mathbb{N}}^{t \in \mathbb{R}_+}$ is characterized as in the following proposition.

Proposition 13. (Dynamics by Redistribution of Asset Holding) If all investors are endowed with only unit of assets, then the dynamics shown in Section 4.2 has the following solutions,

$$\mathcal{M}_L^n(t) = \pi_L \cdot q^n(t), \mathcal{M}_H^n(t) = \pi_H \cdot q^n(t) \text{ for } n \in \mathbb{N},$$

where $q^n(t)$ is presented as below. $\{q_n(t)\}_{n \in \mathbb{N}}^{t \in \mathbb{R}_+}$ adopts the solution as below.

$$\begin{aligned} q_n(t) &= q_n + \sum_{m=1}^{+\infty} A_{nm} e^{-m\lambda_L^+ t} \\ &= q_n + \sum_{k=0}^{+\infty} \left[\frac{(-\theta\lambda_L^- t)^k}{k!} f(n, k) \right] \text{ for } n \in \mathbb{N} \text{ and } t \in \mathbb{R}_+ \end{aligned}$$

where $q_n \equiv \frac{\theta^n}{n \cdot \ln(\frac{1}{1-\theta})}$, and $f(n, k) \equiv \sum_{m=1}^{+\infty} A_{nm} \cdot m^k$ satisfies the following recursive conditions.

$$\begin{aligned} f(n, 0) &= \begin{cases} 1 - q_n & \text{when } n = 1 \\ -q_n & \text{when } n \in \mathbb{N} \setminus \{1\} \end{cases} \\ f(n, k) &= \begin{cases} -\frac{n+1}{\theta} f(n+1, k-1) + n f(n, k-1) & \text{when } n = 1 \text{ and } k \in \mathbb{N} \\ -\frac{n+1}{\theta} f(n+1, k-1) + n f(n, k-1) + \sum_{j=1}^{n-1} f(j, k) & \text{when } n \in \mathbb{N} \setminus \{1\} \text{ and } k \in \mathbb{N} \end{cases} \end{aligned}$$

Using the proposition equips us to reach that

$$\lim_{t \rightarrow \infty} \mathcal{M}_i^n(t) = \mathcal{M}_i^n \text{ for all } n \in \mathbb{N} \text{ and } i \in \{L, H\}.$$

3.5 Asset Liquidity

In addition to bid-ask spread already characterized in Section 2, measurement of asset liquidity also includes the trading volume, trading turnover and liquidity mis-allocation.

3.5.1 Trading Volume and Turnover

By definition, the trading volume and the turnover of asset trading are given as below respectively.

$$\begin{aligned}\mathcal{V}(t) &\equiv \sum_{n=1}^{+\infty} \lambda_L^- n \mu_L^n(t) + \sum_{n=1}^{+\infty} \lambda_H^- n \mu_H^n(t) \\ \mathcal{T}(t) &\equiv \frac{\mathcal{V}(t)}{s}\end{aligned}$$

Then we reach the following corollary by Proposition 1.

Corollary 13. (Trading Volume and Turnover) Given any $(\mu_L^1(t), \mu_H^1(t))$, we have

$$\begin{aligned}\mathcal{V}(t) &= [\mu_L^1(t) \lambda_L^- + \mu_H^1(t) \lambda_H^-] \cdot \left(\frac{s}{1-\theta}\right) \\ \mathcal{T}(t) &= \frac{\mu_L^1(t) \lambda_L^- + \mu_H^1(t) \lambda_H^-}{1-\theta}\end{aligned}$$

In turn, the steady state is formulated as

$$\begin{aligned}\mathcal{V} &= \mu[\lambda_L^- \sum_{n=1}^{+\infty} n \mathcal{M}_L^n + \lambda_H^- \sum_{n=1}^{+\infty} n \mathcal{M}_H^n] = s \cdot (\lambda_L^- \pi_L + \lambda_H^- \pi_H) \\ \mathcal{T} &= \frac{\mathcal{V}}{s} = \lambda_L^- \pi_L + \lambda_H^- \pi_H.\end{aligned}$$

Based on the corollary, we know that the trading volume is positively proportional to s , the total amounts of assets. Besides, both trading volume and turnover increases with $\{\lambda_L^-, \lambda_H^-\}$. The intuition is that, when λ_L^-, λ_H^- increases, more old investors exit and more new investors enter the financial market.

3.5.2 Liquidity Mis-allocation on Asset Holdings

Steady State

The measurement of asset mis-allocation is defined as below.

$$\Delta^{SS} \equiv \frac{\sum_{n=1}^{+\infty} [(x_H - x_L) \cdot n \cdot \mu_L^n]}{x_H \cdot s} = \left(\frac{x_H - x_L}{x_H} \right) \cdot \left(\frac{\lambda_d}{\lambda_u + \lambda_d} \right) = \frac{\Delta x}{x_H} \cdot \pi_L.$$

where the numerator denotes the mis-allocation value of assets held by low-preference investors, the denominator denotes the aggregate value of the assets in the exchange economy.

Transitional Dynamics

First of all, if we are the position of Section 4.2, *i.e.*, investors are subject to unexpected preference shock, then the rate of liquidity mis-allocation is

$$\Delta(t) \equiv \frac{\sum_{n=1}^{+\infty} [(x_H - x_L) \cdot n \cdot \mu_L^n(t)]}{x_H \cdot s} = \frac{\sum_{n=1}^{+\infty} [(x_H - x_L) \cdot n \cdot \mu \cdot \mathcal{M}_L^n(t)]}{x_H \cdot s} = \left(\frac{x_H - x_L}{x_H} \right) \cdot \mathcal{M}_L(t),$$

where $\mathcal{M}_L(t) \equiv [1 - e^{-(\lambda_u + \lambda_d)t}] \cdot \pi_L + e^{-(\lambda_u + \lambda_d)t} \cdot \mathcal{M}_L(0)$. Then we have $\lim_{t \rightarrow \infty} \Delta(t) = \Delta^{SS} \equiv \left(\frac{x_H - x_L}{x_H} \right) \cdot \pi_L$.

Secondly, if investors suffer the shock on redistribution of asset holding, as illustrated in Section 4.3, then the rate of liquidity mis-allocation is

$$\Delta(t) \equiv \frac{\sum_{n=1}^{+\infty} [(x_H - x_L) \cdot n \cdot \mu_L^n(t)]}{x_H \cdot s} = \frac{\sum_{n=1}^{+\infty} [(x_H - x_L) \cdot n \cdot \mu \cdot \mathcal{M}_L^n(t)]}{x_H \cdot s},$$

where $\mathcal{M}_L^n(t) = \pi_L \cdot q^n(t)$ and $q^n(t)$ is characterized in Section 4.3. Again, we have $\lim_{t \rightarrow \infty} \Delta(t) = \Delta^{SS} \equiv \left(\frac{x_H - x_L}{x_H} \right) \cdot \pi_L$.

3.6 Model Extension

This section is devoted to several pieces of extension for the benchmark developed so far. First of all, motivated by Lagos and Rocheteau (2005), we endogenize investor's search intensity. Secondly, we switch from the high-or-low preference to a general

case. Finally, we take into account the endogenous entry decision by market-makers into OTC.

3.6.1 Endogenous Search Intensity

In the spirit of DGP (2005) and Lagos and Rocheteau (2005), we use this part to endogenize investor's search intensity in a tractable way. More specifically, we assume both types of investors have access to a search technology such that the trading intensity is modified from $\{\lambda_i^-, \lambda_i^+\}$ to $\{\gamma_i^- \equiv \lambda_i^- \Omega_i(e_i, n), \gamma_i^+ \equiv \lambda_i^+ \Omega_i(e_i, n)\}$ where $e_i \in \mathbb{R}_+$ denotes the effort of investor with type- i preference. We assume the coefficient of endogenous search intensity, $\Omega(e, n)$, has the following property: i) Ω strictly increases with (e, n) and is homogeneous of degree one with respect to (e, n) , ii) Ω is strictly concave in e , and iii) $\Omega(0, n) = 1$. Then the value functions of Section 3.1 is adjusted as below.

$$\begin{aligned}
rV_L(n, W) &= \max_{e_L \in \mathbb{R}_+} \{n \cdot x_L - e_L + n\gamma_L^- \cdot [V_L(n-1, W+B) - V_L(n, W)] \\
&\quad + n\gamma_L^+ \cdot [V_L(n+1, W-P^f) - V_L(n, W)] \\
&\quad + \lambda_u \cdot [V_H(n, W) - V_L(n, W)] + \frac{\partial V_H(n, W)}{\partial W} \cdot rW\} \\
rV_H(n, W) &= \max_{e_H \in \mathbb{R}_+} \{n \cdot x_H - e_H + n\gamma_H^+ \cdot [V_H(n+1, W-A) - V_H(n, W)] \\
&\quad + n\gamma_H^- \cdot [V_H(n-1, W+P^{fs}) - V_H(n, W)] \\
&\quad - \lambda_d \cdot [V_H(n, W) - V_L(n, W)] + \frac{\partial V_H(n, W)}{\partial W} \cdot rW\}
\end{aligned}$$

where the bid and ask prices as well as those of fire-sale and fire-purchase are determined in a similar way as in Section 3.1. Then we characterize the endogenous search intensity in the following corollary.

Corollary 14. (Endogenous Choice of Search Intensity) For $i \in \{L, H\}$, there exists $v_i \in \mathbb{R}_+$ and $\sigma_i \geq 1$ such that $V_i(n, W) = n \cdot v_i + W$, $\frac{\gamma_i^-}{n \cdot \lambda_i^-} = \frac{\gamma_i^+}{n \cdot \lambda_i^+} = \sigma_i$, where

$\{v_i, \sigma_i\}_{i \in \{L, H\}}$ are jointly characterized as below.

$$\begin{aligned}
rv_L &= x_L - h_L(\sigma_L) + \sigma_L \cdot [\lambda_L^-(1-z) - \lambda_L^+](1-\epsilon)(v_H - v_L) + \lambda_u(v_H - v_L) \\
rv_H &= x_H - h_H(\sigma_H) - \sigma_H \cdot [\lambda_L^- - \lambda_L^+(1-z)]\epsilon(v_H - v_L) - \lambda_d(v_H - v_L) \\
h'_L(\sigma_L) &= [\lambda_L^-(1-z) - \lambda_L^+](1-\epsilon)(v_H - v_L) \\
\sigma_H &= 1
\end{aligned}$$

where $h_i(\cdot) \equiv \omega_i^{-1}(\cdot)$ and $\omega(\cdot) \equiv \Omega(\cdot, 1)$.

Based on the corollary, the equilibrium matching frequency $\{\gamma_i^-, \gamma_i^+\}_{i \in \{L, H\}}$ are obtained in turn. Then we could easily rewrite the whole story of Sections 3 and 4.

3.6.2 General Types on Preference

For the ease of illustration, we follow DGP (2005) to consider only two types of preference over the same assets: high and low. Motivated by Lagos and Rocheteau (2009), we extend our baseline model by allowing for general types of preference. More specifically, we now assume investors could have $I \in \mathbb{N}$ types of preference. For investors with preference-type i , her valuation on one unit of asset is denoted as x_i . Investors may subject to preference shock with Poisson rate $\delta > 0$. Conditioning on a preference shock, investors, currently with preference i , will switch to preference j with probability $\pi_{ij} > 0$. For simplicity, we assume the preference shock are *iid* distributed across investors and over time. Thus we use π_j to denote π_{ij} for all $i \in \mathbb{I} \equiv \{1, 2, \dots, I\}$.¹⁴ By definition, we always have $\sum_{i=1}^I \pi_i = 1$. Without loss of generality, we assume $\{x_i\}_{i \in \mathbb{I}}$ is an increasing sequence.

Based on the above setup, given the idiosyncratic state variables (i, n, W) , the corresponding value function is correspondingly adjusted as below.

$$\begin{aligned}
rV_i(n, W) &= n \cdot x_i + n\lambda_i^- \cdot [V_i(n-1, W+B_i) - V_i(n, W)] + n\lambda_i^+ \cdot [V_i(n+1, W-A_i) - V_i(n, W)] \\
&\quad + \delta \cdot \sum_{j=1}^I \pi_j [V_j(n, W) - V_i(n, W)] + \frac{\partial V_i(n, W)}{\partial W} \cdot rW
\end{aligned}$$

¹⁴In the baseline model with two types of preference, $\delta = \lambda_u + \lambda_d$, $\pi_L = \frac{\lambda_d}{\lambda_u + \lambda_d}$, and $\pi_H = \frac{\lambda_u}{\lambda_u + \lambda_d}$.

In the spirit of Proposition 1, we guess it always holds with general types of preference that $V_i(n, W) = n \cdot v_i + W$ with $\{v_i\}_{i \in \mathbb{I}}$ being an increasing sequence, all of which would be verified later. Moreover, there exists a unique $i^* \in \mathbb{I}$ such that $v_{i^*} \leq M < v_{i^*+1}$. Define the low-preference and high-preference subgroups respectively as $\mathbb{I}_L = \{1, \dots, i^*\}$, $\mathbb{I}_H = \{i^* + 1, \dots, I\}$ such that $\mathbb{I}_L \cap \mathbb{I}_H = \emptyset$ and $\mathbb{I}_L \cup \mathbb{I}_H = \mathbb{I}$. Then we know that investors with preference $i \in \mathbb{I}_L$ are eager to sell. That is, λ_i^- and λ_i^+ denote positive and negative shocks respectively. The scenario is just opposite for investors with preference $i \in \mathbb{I}_H$. Then the bid and ask prices are determined as below.

$$\begin{aligned} A_i(n, W) &= \underset{A \geq 0}{\operatorname{argmax}} [V_i(n+1, W-A) - V_i(n, W)]^{1-z} [A-M]^z \text{ for } i \in \mathbb{I}_H \\ B_j(n, W) &= \underset{B \geq 0}{\operatorname{argmax}} [V_j(n-1, W+B) - V_j(n, W)]^{1-z} [M-B]^z \text{ for } j \in \mathbb{I}_L. \end{aligned}$$

Besides, the prices of fire-sale and fire-purchase are given as follows.

$$P_i^{\text{fire-sale}} = P_j^{\text{fire-purchase}} = M \text{ for } i \in \mathbb{I}_H \text{ and } j \in \mathbb{I}_L.$$

Moreover, as in the baseline model, we pin down the price in the inter-dealer market, M , in the following way.

$$M \in \underset{M' \geq 0}{\operatorname{argmax}} \left\{ \sum_{n \in \mathbb{N}} \sum_{j \in \mathbb{I}_L} [V_j(n-1, W+B) - V_j(n, W) + M-B] \cdot \mu_j^n \right\}^\epsilon \cdot \left\{ \sum_{n \in \mathbb{N}} \sum_{i \in \mathbb{I}_H} [V_i(n+1, W-A) - V_i(n, W) + A-M] \cdot d\mu_i^n \right\}^{1-\epsilon},$$

Finally, by modifying Assumption 1, we make the following assumption in this section for general types of preference.

Assumption 6. $\lambda_i^+ = \lambda_j^+ \equiv \lambda^+$, $\lambda_i^- = \lambda_j^- \equiv \lambda^- \equiv \lambda^+ = \theta \cdot \lambda^-$, for all $i, j \in \mathbb{I}$, where $\theta \in (0, 1)$ is close enough to 1.

Using the proof strategy of Proposition 1 immediately reaches the following corollary.

Corollary 15. (Value Function and Bid-Ask Prices) Under Assumption 3,

1. For investors with preference-type $i \in \mathbb{I} \equiv \{1, 2, \dots, I\}$, n units of financial

assets and W units of liquid assets, the value function is the following form.

$$V_i(n, W) = v_i \cdot n + W,$$

where, given (M, i^*) , $\{v_i\}_{i \in \mathbb{I}}$ is determined as below.

$$v_i = \frac{y_i + \delta \cdot \bar{v}}{r + \delta + \kappa_i},$$

and,

$$\begin{aligned} \kappa_i &\equiv \begin{cases} (1-z)\lambda^- - \lambda^+ & \text{if } i \in \mathbb{I}_L \\ \lambda^- - (1-z)\lambda^+ & \text{if } i \in \mathbb{I}_H \end{cases} \\ y_j &\equiv x_j + \kappa_j \cdot M \\ \bar{v} &\equiv \sum_{j \in \mathbb{I}} \pi_j v_j = \frac{\sum_{j \in \mathbb{I}} (\frac{\pi_j y_j}{r + \delta + \kappa_j})}{1 - \delta \cdot \sum_{j \in \mathbb{I}} (\frac{\pi_j}{r + \delta + \kappa_j})} \\ i^* &= \arg\{i' \in \mathbb{I} : v_{i'} \leq M < v_{i'+1}\} \end{aligned}$$

2. $\{v_i\}_{i \in \mathbb{I}}$ is an increasing sequence.

3. (M, i^*) are jointly determined the following restrictions

$$\begin{aligned} M &= \epsilon \cdot \left(\frac{y_1 + \delta \cdot \bar{v}}{r + \delta + \kappa_1} \right) + (1 - \epsilon) \cdot \left(\frac{y_I + \delta \cdot \bar{v}}{r + \delta + \kappa_I} \right) \\ i^* &= \arg\{i' \in \mathbb{I} : \frac{y_{i'} + \delta \cdot \bar{v}}{r + \delta + \kappa_{i'}} \leq M < \frac{y_{i'+1} + \delta \cdot \bar{v}}{r + \delta + \kappa_{i'+1}}\} \end{aligned}$$

4. The bid and ask prices are formulated as below.

$$\begin{aligned} B_i(n, W) &= B_i \equiv (1-z)M + zv_i \text{ for } i \in \mathbb{I}_L \\ A_j(n, W) &= A_j \equiv (1-z)M + zv_j \text{ for } j \in \mathbb{I}_H \end{aligned}$$

We close this part by solving the distribution of asset holdings with general types of preference. For all preference $i \in \mathbb{I}$, the dynamics of $\mu_i^n(t)$ is formulated as below.

$$d\mu_i^n(t)/dt = \begin{cases} (n+1)\lambda_i^- \cdot \mu_i^{n+1}(t) - n(\lambda_i^- + \lambda_i^+) \cdot \mu_i^n(t) + (n-1)\lambda_i^+ \cdot \mu_i^{n-1}(t) \\ + \delta \cdot \pi_i \sum_{j \neq i} \mu_j^n(t) - \delta(1 - \pi_i) \cdot \mu_i^n(t) & \text{when } n \in \mathbb{N} \setminus \{1\} \\ (n+1)\lambda_i^- \cdot \mu_i^{n+1}(t) - n(\lambda_i^- + \lambda_i^+) \cdot \mu_i^n(t) + \mu_i^{entry}(t) \cdot \eta_i(t) \\ \delta \cdot \pi_i \sum_{j \neq i} \mu_j^n(t) - \delta(1 - \pi_i) \cdot \mu_i^n(t) & \text{when } n = 1 \end{cases}$$

Taking the scenario with $n \geq 2$ for example. Comparing with the dynamics in the case with two types of preference, there is nothing new with the first three items. They are the inflow and outflow of investors with the same preference types. The fourth item denotes the inflow because of the preference shock from other types of investors with the same units of asset holding. The fifth item is the outflow due to the preference shock to the current investor. In steady, $d\mu_i^n(t)/dt = 0$ for all $i \in \mathbb{I}$ and $n \in \mathbb{N}$. Then we have the following corollary.

Corollary 16. (Stationary Size Distribution of Asset Holdings) For all $(i, n) \in \mathbb{I} \times \mathbb{N}$, we have

$$\mathcal{M}_i^n \equiv \frac{\mu_i^n}{\sum_{i \in \mathbb{I}} \sum_{n \in \mathbb{N}} \mu_i^n} = \frac{\pi_i \cdot \theta^n}{n \cdot \ln(\frac{1}{1-\theta})},$$

and thus

$$\begin{aligned} \mathcal{M}^n &= \sum_{i \in \mathbb{I}} \mathcal{M}_i^n = \frac{\theta^n}{n \cdot \ln(\frac{1}{1-\theta})} \\ \mathcal{M}_i &= \sum_{n \in \mathbb{N}} \mathcal{M}_i^n = \pi_i \end{aligned}$$

Similar to the results of Proposition 2, the size distribution of asset holdings is still analytically tractable even with general types of preference. Meanwhile, we still preserve the pattern of logarithmic distribution even though restriction on $\{0, 1\}$ -asset holding is removed.

3.6.3 Free Entry of Market-makers

We could endogenize the trading frequency as what Lagos and Rocheteau (2009) does. Denote $\kappa > 0$ as the fixed cost to become a dealer in OTC. Denote $\lambda_i^+ \cdot \varphi(\cdot)$ and $\lambda_i^- \cdot \varphi(\cdot)$ as the matching intensity in OTC. Then the free entry condition of

dealers in either the seller or buyer side is written as below, which in turn pins down $\{\mu_B^d, \mu_S^d\}$.

$$\sum_{n=1}^{\infty} \mathcal{M}_L^n \cdot \frac{n\lambda_L^- \varphi(\mu_S^d)}{\mu_S^d} (M - B) = \kappa = \sum_{n=1}^{\infty} \mathcal{M}_H^n \cdot \frac{n\lambda_H^+ \varphi(\mu_B^d)}{\mu_B^d} (A - M)$$

where $\mathcal{M}_i^n = \frac{\pi_i \theta^{n-1}}{n \cdot \ln(\frac{1}{1-\theta})}$, $i \in \{L, H\}$. Moreover, based on Proposition 1, $\{M - B, A - M\}$ are accordingly adjusted as below.

$$\begin{aligned} M - B &= (1 - \epsilon)(v_H - v_L) \\ A - M &= \epsilon(v_H - v_L) \end{aligned}$$

and

$$v_H - v_L = \frac{x_H - x_L}{r + \lambda_u + \lambda_d + \{(1 - \epsilon)[\lambda_H^- \cdot \varphi(\mu_S^d) - \lambda_H^+ (1 - z) \cdot \varphi(\mu_B^d)] + \epsilon[\lambda_L^- (1 - z) \cdot \varphi(\mu_S^d) - \lambda_L^+ \cdot \varphi(\mu_B^d)]\}}.$$

Then $\{\mu_B^d, \mu_S^d\}$ can be easily pinned down and in turn we can recover the endogenous trading intensity.

3.7 Conclusion

This paper uses the birth-and-death process in the literature of firm dynamics to analytically characterize the non-stationary life-cycle asset trading in OTC, which involves in search frictions and bargaining. Although the indivisibility restriction on asset holding in DGP (2005) is removed, we still manage to obtain explicit solutions on the value functions, bid and ask prices and the size distribution of asset holding in both steady state and transitional dynamics. The mis-allocation rate of asset liquidity is shown to be related to the speed of preference shock as well as to that of trading intensity. Moreover, we fully characterize the life-cycle pattern of asset holding by each cohort of investors.

Our model has several pieces of testable implications. First of all, the stationary size distribution of asset holding follows a logarithmic pattern. Secondly, the life cycle of asset holding by each cohort conforms to a geometric distribution while the size distribution of asset holding in each cross-section follows a logarithmic pattern.

Thirdly, the average growth rate of asset holding is irrelevant to the size of current asset holdings. Meanwhile, the volatility of growth rate of asset holding decreases with the size of current asset holdings. That is, we reach the results on Gibrat's law on asset trading in OTC.

Throughout this paper we mainly focus on the implication of search frictions for life cycle as well as the size distribution of asset holdings. Thus we have deliberately assumed away information frictions from the context. However, OTC is sometimes called opaque market.¹⁵ It is not only due to search frictions and bargaining, but also because of information frictions. Therefore it could be fruitful to introduce into the model adverse selection on asset quality or information asymmetry on investor's private evaluation of the same assets. Additionally, the framework developed in our paper produces several pieces of testable implications. Empirical tests on these predictions could be put in our research agenda in the near future.

3.8 Appendix : Omitted Proofs in the Context

Proof of Proposition 1

Proof. Conjecture that, there exists pairwise values (v_L, v_H) such that, for $i = L, H$, we have

$$V_i(n, W) = v_i \cdot n + W.$$

In turn, given (M, v_L, v_H) , the bid price(s) $B(n, W)$ could be simplified as below.

$$\begin{aligned} B(n, W) &= \operatorname{argmax}[V_L(n-1, W+B) - V_L(n, W)]^{1-z}[M-B]^z \\ &= \operatorname{argmax}[B - v_L]^{1-z}[M-B]^z \\ &= zv_L + (1-z)M. \end{aligned}$$

Similarly, the ask prices $A(n, W)$ could be solved as

$$A(n, W) = zv_H + (1-z)M.$$

¹⁵See Zhu (2012) among others for the discussion on opaqueness of OTC.

As a result, we have

$$Spread = B - A = z(v_H - v_L)$$

Using the conjecture that $V_i(n, W) = v_i \cdot n + W$ again for the formula on how to determine M , the inter-dealer market price, and then we have

$$M = \epsilon v_1 + (1 - \epsilon)v_H.$$

Substituting the above formula on M into $B(n, W)$ and $A(n, W)$ yields that

$$\begin{aligned} B &= v_L + (1 - z)\epsilon(v_H - v_L) \\ A &= v_H - (1 - z)(1 - \epsilon)(v_H - v_L) \end{aligned}$$

Substituting all of the above-mentioned results into the original value functions for investors with preference L and H yields the following simultaneous equations.

$$\begin{aligned} rv_L &= x_L + \lambda_L^-(1 - z)\epsilon(v_H - v_L) - \lambda_L^+(M - v_L) + \lambda_u(v_H - v_L) \\ rv_H &= x_H + \lambda_H^+(1 - z)(1 - \epsilon)(v_H - v_L) - \lambda_H^-(v_H - M) - \lambda_d(v_H - v_L) \end{aligned}$$

Taking difference between the above two equations yields

$$\{r + \lambda_u + \lambda_d + (1 - \epsilon)[\lambda_H^- - \lambda_H^+(1 - z)] + \epsilon[\lambda_L^-(1 - z) - \lambda_L^+]\}(v_H - v_L) = x_H - x_L$$

and thus

$$v_H - v_L = \frac{x_H - x_L}{r + \lambda_u + \lambda_d + (1 - \epsilon)[\lambda_H^- - \lambda_H^+(1 - z)] + \epsilon[\lambda_L^-(1 - z) - \lambda_L^+]}$$

Using Assumption 2 immediately implies that $v_H - v_L > 0$. Moreover, substituting the above results into either of the simultaneous equations could recover v_H and v_L . Finally, using Assumptions 1 and 2 together suggests that $v_H < \frac{x_H}{r}$ and $v_L > \frac{x_L}{r}$.

□

Proof of Proposition 2

Proof. For the first part.

We have $d\mu_i^n(t)/dt = 0$ in steady state and thus all time scripts are removed. Then the dynamic system in Section 3.3 is simplified as below.

$$\lambda_d \cdot \mu_H^n = \lambda_u \cdot \mu_L^n,$$

and for $i \in \{L, H\}$, we have

$$\begin{aligned} (n+1)\lambda_i^- \mu_i^{n+1} - n(\lambda_i^- + \lambda_i^+) \mu_i^n + (n-1)\lambda_i^+ \mu_i^{n-1} &= 0 \quad \text{for } n \in \mathbb{N}/\{1\} \\ (n+1)\lambda_i^- \mu_i^{n+1} - n(\lambda_i^- + \lambda_i^+) \mu_i^n + \mu_i^{entry} \eta_L &= 0 \quad \text{for } n = 1 \end{aligned}$$

As a result, we have

$$\begin{aligned} (n+1)\lambda_i^- \mu_i^{n+1} &= n\lambda_i^+ \mu_i^n \quad \text{for } n \in \mathbb{N}/\{1\} \\ \lambda_i^- \mu_i^n &= \mu_i^{entry} \eta_L \quad \text{for } n = 1 \end{aligned}$$

Thus we have

$$\mu_i^n = \frac{\mu_i^1 \theta^{n-1}}{n} \quad (\#).$$

Using Assumption 1 simplifies Eq. (3.1) as

$$(1-\theta)\lambda_L^- \sum_{n=1}^{+\infty} n\mu_L^n(t) + (1-\theta)\lambda_H^- \sum_{n=1}^{+\infty} n\mu_H^n(t) = \sum_{i \in \{L, H\}} \eta_i(t)$$

Additionally, combining Eq. (3.1) and Eq. (3.2) suggests that

$$\begin{aligned} \sum_{n=1}^{+\infty} n\mu_L^n(t) &= \frac{\sum_{i \in \{L, H\}} \eta_i(t) - s(1-\theta)\lambda_H^-}{(1-\theta)(\lambda_L^- - \lambda_H^-)} \\ \sum_{n=1}^{+\infty} n\mu_H^n(t) &= \frac{s(1-\theta)\lambda_L^- - \sum_{i \in \{L, H\}} \eta_i(t)}{(1-\theta)(\lambda_L^- - \lambda_H^-)} \end{aligned}$$

To make sure the above two equations well-defined, we must have $s \in (\frac{\sum_{i \in \{L, H\}} \eta_i(t)}{(1-\theta)\lambda_L^-}, \frac{\sum_{i \in \{L, H\}} \eta_i(t)}{(1-\theta)\lambda_H^-})$ hold in equilibrium. We use guess-and-verify to show that the above internal restrictions is always held in equilibrium.

Substituting (#) into the above equations yields

$$\mu_L^1 = s(1 - \theta)\pi_L, \text{ and } \mu_H^1 = s(1 - \theta)\pi_H,$$

where $\pi_L \equiv \frac{\lambda_d}{\lambda_u + \lambda_d}$ and $\pi_H = 1 - \pi_L$. In turn, the total measure of asset holders in the economy is

$$\mu \equiv \sum_{n=1}^{+\infty} (\mu_L^n + \mu_H^n) = (1 + \frac{\lambda_u}{\lambda_d}) \cdot \mu_L^1 \cdot \sum_{n=1}^{+\infty} (\frac{\theta^{n-1}}{n}) = \frac{s(1 - \theta)}{\theta} \cdot \ln(\frac{1}{1 - \theta})$$

Moreover, since we normalize that $\mu_L^{entry} = \mu_H^{entry} = 1$, the endogenous entry rate is pinned down as below.

$$\begin{aligned} \eta_L &= \lambda_L^- \mu_L^1 = s(1 - \theta)\lambda_L^- \pi_L. \\ \eta_H &= \lambda_H^- \mu_H^1 = s(1 - \theta)\lambda_H^- \pi_H \end{aligned}$$

Finally, we could check that $s \in (\frac{\sum_{i \in \{L, H\}} \eta_i(t)}{(1 - \theta)\lambda_L^-}, \frac{\sum_{i \in \{L, H\}} \eta_i(t)}{(1 - \theta)\lambda_H^-})$ is satisfied indeed.

For the second part.

According to the dynamic system in Section 3.3,

$$n_i(t + \varepsilon) = \begin{cases} n_i(t) + 1 & w.p. \lambda_i^+ n_i(t) \varepsilon \\ n_i(t) - 1 & w.p. \lambda_i^- n_i(t) \varepsilon \\ n_i(t) & o.w. \end{cases}$$

As result, we have

$$\mathbb{E}[\frac{dn_i(t)/dt}{n_i(t)}] = \lim_{\varepsilon \rightarrow 0} \frac{\mathbb{E}[n_i(t + \varepsilon) - n_i(t)]}{\varepsilon \cdot n_i(t)} = \lambda_i^+ - \lambda_i^-$$

Similarly, we have

$$Var[\frac{dn_i(t)/dt}{n_i(t)}] = \frac{\lambda_i^+ + \lambda_i^-}{n_i(t)}.$$

□

Proof of Proposition 3

Proof. The first case:

Given our initial conditions, we guess that in the dynamic transition path, we always have

$$p_H^n(t; n_0 = 1) = \left(\frac{\lambda_u}{\lambda_d}\right) \cdot p_L^n(t; n_0 = 1) \text{ for } n \in \mathbb{N} \cup \{0\}.$$

Since it is always held that

$$\sum_{n=0}^{\infty} p_H^n(t; n_0 = 1) + \sum_{n=0}^{\infty} p_L^n(t; n_0 = 1) = 1.$$

Combining the above two conditions suggests that

$$\sum_{n=0}^{\infty} p_L^n(t; n_0 = 1) = \frac{\lambda_d}{\lambda_u + \lambda_d} = \pi_L.$$

Now, suggested by Klette and Kortum (2004), we define probability-generating function as below.

$$\mathcal{G}_L(\alpha, t) = \sum_{n=0}^{\infty} p_L^n(t) \cdot \alpha^n.$$

Then we have

$$\begin{aligned} \frac{\partial \mathcal{G}_L(\alpha, t)}{\partial \alpha} &= \sum_{n=1}^{\infty} n \cdot p_L^n(t) \cdot \alpha^{n-1} \\ \frac{\partial \mathcal{G}_L(\alpha, t)}{\partial t} &= \frac{dp_L^0(t)}{dt} + \sum_{n=1}^{\infty} \frac{dp_L^n(t)}{dt} \cdot \alpha^n \end{aligned}$$

Combining the above two equations with the dynamic system in Section 4.1 suggests that $\mathcal{G}_L(\alpha, t)$ satisfies the following partial-differential equation (PDE).

$$\frac{\partial \mathcal{G}_L(\alpha, t)}{\partial t} = [\lambda_L^+ \cdot \alpha^2 - (\lambda_L^- + \lambda_L^+) \cdot \alpha + \lambda_L^-] \cdot \frac{\partial \mathcal{G}_L(\alpha, t)}{\partial \alpha}.$$

Following Narendra and Richter-Dyn (1974), we set up the initial condition as in Proposition 3. By definition of $\mathcal{G}_L(\alpha, t)$, we have

$$\mathcal{G}_L(\alpha, 0; n_0) = \sum_{n=0}^{\infty} p_L^n(t; n_0) \cdot \alpha^n = \pi_L \cdot \alpha^n.$$

Combining this initial condition with the above PDE produces the analytic solution

on $\mathcal{G}_L(\alpha, t)$ as below.

$$\mathcal{G}_L(\alpha, t; n_0) = \sum_{n=0}^{\infty} p_L^n(t; n_0) \cdot \alpha^n = \left[\frac{\lambda_L^-(\alpha - 1)e^{-(\lambda_L^- - \lambda_L^+)t} - (\lambda_L^+ \alpha - \lambda_L^-)}{\lambda_L^+(\alpha - 1)e^{-(\lambda_L^- - \lambda_L^+)t} - (\lambda_L^+ \alpha - \lambda_L^-)} \right]^{n_0}.$$

Then, the Taylor series expansion of $\mathcal{G}_L(\alpha, t; n_0)$ around $\alpha = 0$ yields $p_L^n(t; n_0)$ as below.

$$\begin{aligned} p_L^n(t; n_0) &= \frac{1}{n!} \frac{\partial^n \mathcal{G}_L(\alpha, t; n_0)}{\partial \alpha^n} \Big|_{\alpha=0}, \quad \text{for } n \in \mathbb{N} \\ p_L^0(t; n_0) &= \mathcal{G}_L(0, t; n_0) \end{aligned}$$

In particular, when $n_0 = 1$, we reach the results on $\{p_L^n(t; n_0)\}_{n \in \mathbb{N} \cup \{0\}}$ in Proposition 3. As a result, $\{p_H^n(t; n_0)\}_{n \in \mathbb{N} \cup \{0\}}$ is by recovered by revoking the relationship of $p_H^n(t; n_0 = 1) = (\frac{\lambda_u}{\lambda_d}) \cdot p_L^n(t; n_0 = 1)$ for all $n \in \mathbb{N} \cup \{0\}$.

Finally, at time t , conditioning on still staying in OTC, the (truncated) size distribution of investors with preference type i from the zero-cohort conforms to a geometric distribution as below.

$$\frac{p_L^n(t; n_0 = 1)}{1 - p_L^0(t; n_0 = 1)} = [1 - \sigma(t)]\sigma(t)^{n-1}, \quad \text{for } i \in \{L, H\} \text{ and } n \in \mathbb{N}.$$

The second case:

Since $\lambda_L^- = \lambda_H^- = \lambda^-$ and $\lambda_L^+ = \lambda_H^+ = \lambda^+$, combining the dynamics of $p_L^n(t; n_0)$ and $p_H^n(t; n_0)$ yields the dynamics for $p^n(t; n_0) = p_L^n(t; n_0) + p_H^n(t; n_0)$ as below.

$$dp^n(t; n_0)/dt = \begin{cases} (n+1)\lambda^- \cdot p^{n+1}(t; n_0) - n(\lambda^- + \lambda^+) \cdot p^n(t; n_0) + (n-1)\lambda^+ \cdot p^{n-1}(t; n_0) & \text{when } n \in \mathbb{N} \setminus \{1\} \\ (n+1)\lambda^- \cdot p^{n+1}(t; n_0) - n(\lambda^- + \lambda^+) \cdot p^n(t; n_0) & \text{when } n = 1 \\ (n+1)\lambda^- \cdot p^{n+1}(t; n_0) & \text{when } n = 0 \end{cases}$$

Then we can solve $p^n(t; n_0)$ by resorting to the procedure proposed in the first case for solving $p_L^n(t; n_0)$.

□

Proof of Corollary 1

Proof. Given the initial asset holding as $n_0 = 1$, define $F_L(t, n_0)$ as the CDF of investors with preference type L exiting OTC before time t . Then we have

$$\mathbb{E}[T_L] = \int_0^{+\infty} t \cdot dF_L(t; n_0) = \int_0^{+\infty} [1 - F_L(t; n_0)] dt,$$

where the second equation is obtained by using integration by parts.

Notice that $F_L(t; n_0) = p_L^0(t; n_0) / \sum_{n=0}^{\infty} p_L^n(t; n_0)$ due to law of large numbers. Using the analytic results of $p_L^0(t; n_0)$ in Proposition 3 yields

$$\mathbb{E}[T_L] = \int_0^{+\infty} [p_L^0(t; n_0 = 1) / \sum_{n=0}^{\infty} p_L^n(t; n_0)] dt = \frac{\ln(\frac{\lambda_L^-}{\lambda_L^- - \lambda_L^+})}{\lambda_L^+}.$$

We can then get $\mathbb{E}[T_H]$ in a similar way. Finally, since $\frac{\lambda_L^+}{\lambda_L^-} = \frac{\lambda_H^+}{\lambda_H^-} = \theta$, we can also express $\mathbb{E}[T_i]$ in terms of θ .

□

Proof of Proposition 4

Proof. Given the initial condition, we guess that, for all $(t, n) \in \mathbb{R}_+ \times \mathbb{N}$, dynamic system in Section 4.2 is simplified as below.

$$\begin{aligned} d\mathcal{M}_L^n(t)/dt &= \lambda_d \cdot \mathcal{M}_H^n(t) - \lambda_u \cdot \mathcal{M}_L^n(t) \\ d\mathcal{M}_H^n(t)/dt &= -\lambda_d \cdot \mathcal{M}_H^n(t) + \lambda_u \cdot \mathcal{M}_L^n(t) \end{aligned}$$

and

$$(n+1)\lambda_i^- \cdot \mathcal{M}_i^{n+1}(t) - n(\lambda_i^- + \lambda_i^+) \cdot \mathcal{M}_i^n(t) + (n-1)\lambda_i^+ \cdot \mathcal{M}_i^{n-1}(t) = 0 \quad \text{when } n \in \mathbb{N} \setminus \{1\} \quad (A1)$$

$$(n+1)\lambda_i^- \cdot \mathcal{M}_i^{n+1}(t) - n\lambda_i^+ \cdot \mathcal{M}_i^n(t) = 0 \quad \text{when } n = 1 \quad (A2)$$

Denote $\mathcal{M}_i(t) = \sum_{n=1}^{\infty} \mathcal{M}_i^n(t)$. Then the above simplified dynamic system implies

$$\begin{aligned} d\mathcal{M}_L(t)/dt &= \lambda_d \cdot \mathcal{M}_H(t) - \lambda_u \cdot \mathcal{M}_L(t) \\ d\mathcal{M}_H(t)/dt &= -\lambda_d \cdot \mathcal{M}_H(t) + \lambda_u \cdot \mathcal{M}_L(t) \end{aligned}$$

Combining the above two equations yields that

$$d\mathcal{M}_L(t)/dt + d\mathcal{M}_H(t)/dt = 0,$$

and thus $\mathcal{M}_H(t) = 1 - \mathcal{M}_L(t)$. In turn, we have

$$d\mathcal{M}_L(t)/dt = \lambda_d \cdot [1 - \mathcal{M}_L(t)] - \lambda_u \cdot \mathcal{M}_L(t) = \lambda_d - (\lambda_u + \lambda_d) \cdot \mathcal{M}_L(t)$$

Then the ordinary differential equation (ODE) on $\mathcal{M}_L(t)$ admits the solution as below.

$$\mathcal{M}_L(t) \equiv [1 - e^{-(\lambda_u + \lambda_d)t}] \cdot \pi_L + e^{-(\lambda_u + \lambda_d)t} \cdot \mathcal{M}_L(0),$$

and in turn,

$$\mathcal{M}_H(t) = 1 - \mathcal{M}_L(t) = [1 - e^{-(\lambda_u + \lambda_d)t}] \cdot \pi_H + e^{-(\lambda_u + \lambda_d)t} \cdot \mathcal{M}_H(0).$$

Finally, by using equations (A1) and (A2) and following the procedure of Proposition 1, we can obtain that

$$\mathcal{M}_i^n(t)/\mathcal{M}_i(t) = \frac{\theta^n}{n \cdot \ln(\frac{1}{1-\theta})}.$$

□

Proof of Proposition 5

Proof. The dynamic system in Section 4.2 is essentially an infinite-dimensional linear dynamic system. Given the initial condition, we follow Bryson and Ho (1975) to get the solutions in the form shown in this proposition.

□

Proof of Corollary 2

Proof. First, by definition, the trading volume at each time t is,

$$\begin{aligned}
\mathcal{V}(t) &\equiv \sum_{n=1}^{+\infty} \lambda_L^- n \mu_L^n(t) + \sum_{n=1}^{+\infty} \lambda_H^- n \mu_H^n(t) \\
&= \lambda_L^- \frac{\sum_{i \in \{L, H\}} \eta_i(t) - s(1-\theta) \lambda_H^-}{(1-\theta)(\lambda_L^- - \lambda_H^-)} + \lambda_H^- \frac{s(1-\theta) \lambda_L^- - \sum_{i \in \{L, H\}} \eta_i(t)}{(1-\theta)(\lambda_L^- - \lambda_H^-)} \\
&= \frac{\sum_{i \in \{L, H\}} \eta_i(t)}{1-\theta} = [\mu_L^1(t) \lambda_L^- + \mu_H^1(t) \lambda_H^-] \cdot \left(\frac{s}{1-\theta}\right).
\end{aligned}$$

where the second equation is derived by using Proposition 1.

Secondly, the turnover of the asset is defined as below.

$$\begin{aligned}
\mathcal{T}(t) &= \frac{\mathcal{V}(t)}{s} = \left(\frac{\mu}{s}\right) \cdot [\lambda_L^- \sum_{n=1}^{+\infty} n \mathcal{M}_L^n(t) + \lambda_H^- \sum_{n=1}^{+\infty} n \mathcal{M}_H^n(t)] \\
&= \left(\frac{1-\theta}{\theta}\right) \cdot \ln\left(\frac{1}{1-\theta}\right) \cdot [\lambda_L^- \sum_{n=1}^{+\infty} n \mathcal{M}_L^n(t) + \lambda_H^- \sum_{n=1}^{+\infty} n \mathcal{M}_H^n(t)] \\
&= \frac{\mu_L^1(t) \lambda_L^- + \mu_H^1(t) \lambda_H^-}{1-\theta}.
\end{aligned}$$

Finally, in steady state, we have

$$\begin{aligned}
\mathcal{V} &= \mu [\lambda_L^- \sum_{n=1}^{+\infty} n \mathcal{M}_L^n + \lambda_H^- \sum_{n=1}^{+\infty} n \mathcal{M}_H^n] = s \cdot (\lambda_L^- \pi_L + \lambda_H^- \pi_H) \\
\mathcal{T} &= \frac{\mathcal{V}}{s} = \lambda_L^- \pi_L + \lambda_H^- \pi_H.
\end{aligned}$$

□

Proof of Corollary 3

Proof. First of all, in spirit of Proposition 1, we guess and then would later verify that, there exists $v_i \in \mathbb{R}_+$ such that $V_i(n, W) = n \cdot v_i + W$. Secondly, since $\Omega(e, n)$ is homogeneous of degree one in (e, n) , we have $\gamma_i^- = \lambda_i^- \Omega_i(e_i, n) = \lambda_i^- n \cdot \Omega_i(\frac{e_i}{n}, 1) \equiv \lambda_i^- n \cdot \omega_i(\frac{e_i}{n})$, where $\omega(\cdot)$ is a strictly increasing and strictly concave function. Denoting $h_i(\cdot) \equiv \omega_i^{-1}(\cdot)$, we have $e_i = n \cdot h_i(\frac{\gamma_i^-}{n \cdot \lambda_i^-}) = n \cdot h_i(\frac{\gamma_i^+}{n \cdot \lambda_i^+})$. Since $\gamma_i^- \equiv \lambda_i^- \Omega_i(e_i, n)$, $\gamma_i^+ \equiv \lambda_i^+ \Omega_i(e_i, n)$, we have $\frac{\gamma_i^-}{\lambda_i^-} = \frac{\gamma_i^+}{\lambda_i^+}$. Substituting e_i into the original value functions yields the following two

conditions.

$$\begin{aligned} rv_L &= \max_{\sigma'_L \geq 1} \{x_L - h_L(\sigma'_L) + \sigma'_L[\lambda_L^-(1-z) - \lambda_L^+](1-\epsilon)(v_H - v_L)\} + \lambda_u(v_H - v_L) \\ rv_H &= \max_{\sigma'_H \geq 1} \{x_H - h_H(\sigma'_H) - \sigma'_H[\lambda_L^- - \lambda_L^+(1-z)]\epsilon(v_H - v_L)\} - \lambda_d(v_H - v_L) \end{aligned}$$

First order condition on $\{\sigma_L, \sigma_H\}$ are in turn obtained as below.

$$\begin{aligned} h'_L(\sigma_L) &= [\lambda_L^-(1-z) - \lambda_L^+](1-\epsilon)(v_H - v_L) \\ \sigma_H &= 1 \end{aligned}$$

The corner solution on σ_H is reached because of Assumption 1. Pooling the the above condition finishes the proof of this corollary. □

Proof of Corollary 4

Proof. Again, we use guess-and-verify for the form of value functions, *i.e.*, we guess that

$$V_i(n, W) = n \cdot v_i + W.$$

Substituting it into the original value function in Section 6.2, we know that $\{v_1, \dots, v_I; i^*, M\}$ are jointly determined as below.

$$\begin{aligned} rv_i &= x_i + \kappa_i(M - v_i) + \delta \cdot \sum_{j=1}^I \pi_j(v_j - v_i) \\ M &= \epsilon v_1 + (1 - \epsilon)v_I \\ i^* &= \arg_{i \in \mathbb{I}} \{v_{i^*} \leq M < v_{i^*+1}\} \end{aligned}$$

where

$$\kappa_i \equiv \begin{cases} (1-z)\lambda_i^- - \lambda_i^+ & \text{if } i \in \mathbb{I}_L \\ \lambda_i^- - (1-z)\lambda_i^+ & \text{if } i \in \mathbb{I}_H \end{cases}$$

By Assumption 3, κ_i can be rewritten as below.

$$\kappa_i \equiv \begin{cases} (1-z)\lambda^- - \lambda^+ & \text{if } i \in \mathbb{I}_L \\ \lambda^- - (1-z)\lambda^+ & \text{if } i \in \mathbb{I}_H \end{cases}$$

Given (M, i^*) , $\{v_i\}_{i \in \mathbb{I}}$ can be solved as below.

$$v_i = \frac{y_i + \delta \cdot \bar{v}}{r + \delta - \kappa_i},$$

where

$$\begin{aligned} y_j &\equiv x_j + \kappa_j \cdot M \\ \bar{v} &\equiv \sum_{j \in \mathbb{I}} \pi_j v_j = \frac{\sum_{j \in \mathbb{I}} (\frac{\pi_j y_j}{r + \delta + \kappa_j})}{1 - \delta \cdot \sum_{j \in \mathbb{I}} (\frac{\pi_j}{r + \delta + \kappa_j})}. \end{aligned}$$

Due to Assumption 3, θ is close enough to one and thus κ_i is close enough to κ_j , for all $i \in \mathbb{I}_L$ and $j \in \mathbb{I}_H$. Besides, we already know that $\{x_i\}_{i \in \mathbb{I}}$, then the result that $v_i = \frac{y_i + \delta \cdot \bar{v}}{r + \delta - \kappa_i}$ immediately suggests that $v_i > v_{i-1}$ holds for all $i \in \mathbb{I}$.

Since $M = \epsilon v_1 + (1 - \epsilon)v_I$, we have

$$M = \epsilon \cdot \left(\frac{y_1 + \delta \cdot \bar{v}}{r + \delta + \kappa_1} \right) + (1 - \epsilon) \cdot \left(\frac{y_I + \delta \cdot \bar{v}}{r + \delta + \kappa_I} \right).$$

Moreover, substituting v_i into the definition of i^* yields

$$i^* = \arg\{i' \in \mathbb{I} : \frac{y_{i'} + \delta \cdot \bar{v}}{r + \delta + \kappa_{i'}} \leq M < \frac{y_{i'+1} + \delta \cdot \bar{v}}{r + \delta + \kappa_{i'+1}}\}.$$

Combining the above two equation pins down (M, i^*) , which in turn can be used to recover $\{v_i\}_{i \in \mathbb{I}}$. □

Proof of Corollary 5

Proof. We have $d\mu_i^n(t)/dt = 0$ in steady state and thus all time scripts are removed. Then for for all $i \in \mathbb{I}$ the dynamic system in Section 3.3 is simplified as below.

$$\pi_i \cdot \sum_{j \neq i} \mu_j^n = (1 - \pi_i) \cdot \mu_i^n \text{ for } n \in \mathbb{N},$$

and

$$\begin{aligned} (n+1)\lambda_i^- \mu_i^{n+1} - n(\lambda_i^- + \lambda_i^+) \mu_i^n + (n-1)\lambda_i^+ \mu_i^{n-1} &= 0 \text{ for } n \in \mathbb{N}/\{1\} \\ (n+1)\lambda_i^- \mu_i^{n+1} - n(\lambda_i^- + \lambda_i^+) \mu_i^n + \mu_i^{entry} \eta_L &= 0 \text{ for } n = 1 \end{aligned}$$

The first equation immediately implies that

$$\mu_i^n = \pi_i \mu^n,$$

where $\mu^n \equiv \sum_{j=1}^{\infty} \mu_j^n$.

The second equation suggests that

$$\begin{aligned} (n+1)\lambda_i^- \mu_i^{n+1} &= n\lambda_i^+ \mu_i^n \text{ for } n \in \mathbb{N}/\{1\} \\ \lambda_i^- \mu_i^n &= \mu_i^{entry} \eta_L \text{ for } n = 1 \end{aligned}$$

Thus we have

$$\mu_i^n = \frac{\mu_i^1 \cdot \theta^{n-1}}{n}.$$

Then we are done by following the argument in Proposition 1, which considers the scenario with two types of investor preference.

□

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